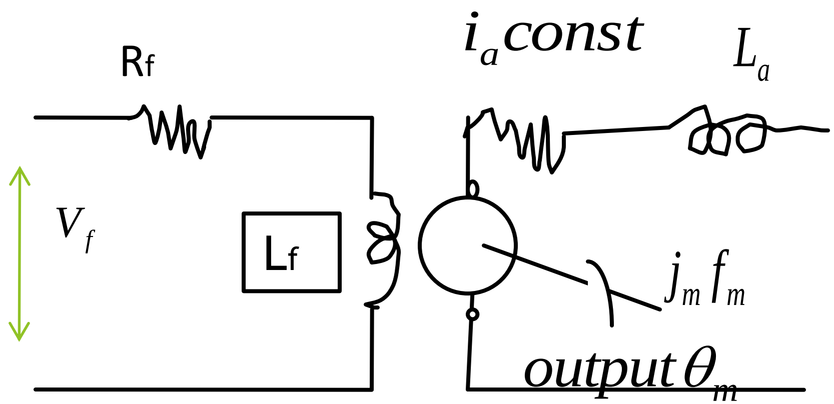
SERVO MOTOR
FIELD CONTROLLED
D.C.MOTOR



Jm moment of inertia
Fm coefficient of friction
θm angular shift
ωm angular velocity
Tm motor torque
Kf is motor torque constant

Field controlled d.c. motor

$$T_M \alpha i_f$$

$$T_M = K_f i_f$$

$$V_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$T_M = K_f I_f$$

$$T_M = J_m \frac{d^2\theta_m}{dt^2} + f_m \frac{d\ \theta_m}{dt}$$

$$T_M = J_m \frac{d\omega_m}{dt} + f_m \omega_m$$

$$V_f(s) = R_f I_f(s) + sL_f I_f(s)$$

$$T_M = K_f I_f(s)$$

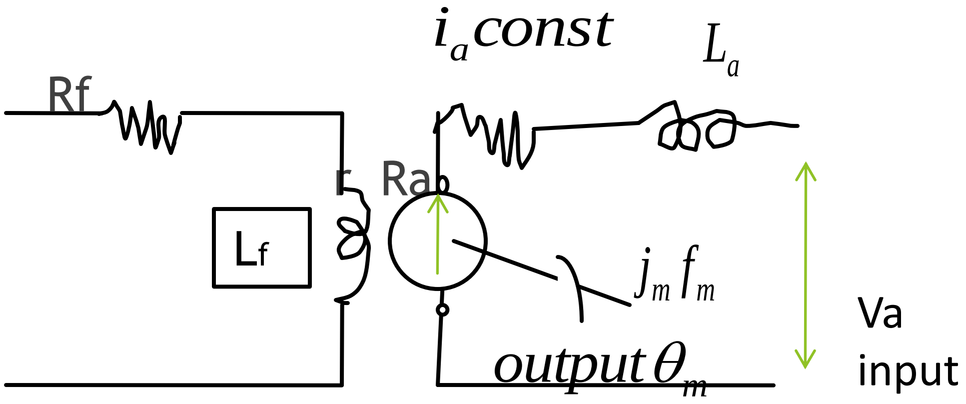
$$T_M = s^2 J_m \theta_m(s) + s f_m \theta_m(s)$$

$$T_M = sJ_m \omega_m(s) + f_m \omega_m(s)$$

$$\frac{\theta_m(s)}{V_f(s)} = \frac{K_f}{s(R_f + sL_f)(sJ_m + f_m)}$$

$$\frac{\omega_m(s)}{V_f(s)} = \frac{K_f}{(R_f + sL_f)(sJ_m + f_m)}$$

SERVO MOTOR
Armature CONTROLLED D.C.MOTOR



Armature controlled DC motor

$$T_M \alpha i_a$$

$$T_M = K_T i_a$$

K_T is the motor torque

$$e_b \alpha \omega_m \because \omega_m = \frac{d\theta}{dt}$$

$$e_b = K_b \frac{d\theta_m}{dt}$$

K_b is back emf

$$V_a - e_b = R_a i_a + L_a \frac{di_a}{dt}$$

$$e_b = K_b \frac{d\ \theta_m}{dt}$$

$$e_b = K_b \omega_m$$

$$T_M = K_T i_a$$

$$T_M = J_m \frac{d^2\theta_m}{dt^2} + f_m \frac{d\ \theta_m}{dt}$$

$$T_M = J_m \frac{d\omega_m}{dt} + f_m \omega_m$$

$$V_a(s) - E_b(s) = R_a I_a(s) + sL_a I_a(s)$$

$$E_b(s) = sK_b\theta_m$$

$$E_b(s) = K_b\omega_m(s)$$

$$T_M(s) = K_T I_a(s)$$

$$T_M(s) = s^2 J_m \theta_m(s) + sf_m\theta_m(s)$$

$$T_M(s) = sJ_m\omega_m(s) + f_m\omega_m(s)$$

$$G(s) = \frac{K_T}{s(R_a + sL_a)(sJ_m + f_m)}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{\dfrac{K_T}{s(R_a + sL_a)(sJ_m + f_m)}}{1 + \dfrac{K_T}{s(R_a + sL_a)(sJ_m + f_m)} sK_b}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_T}{s(R_a + sL_a)(sJ_m + f_m) + sK_b K_T}$$

If the armature inductance is negelected

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_T}{sR_a(sJ_m + f_m) + sK_bK_T}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_T}{s(sR_aJ_m + f_mR_a + K_bK_T)}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{\left[\dfrac{K_T}{(f_mR_a + K_bK_T)}\right]}{s\left[\dfrac{sR_aJ_m}{f_mR_a + K_bK_T} + 1\right]}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_m}{s(1 + sT_m)}$$

$$where K_m = \frac{K_T}{(f_mR_a + K_bK_T)}$$

$$T_m = \frac{R_aJ_m}{(f_mR_a + K_bK_T)}$$

$$\omega_m(s) = s\theta_m(s)$$

$$\frac{\omega_m(s)}{V_a(s)} = \frac{K_m}{(1 + sT_m)}$$

Relation between torque and back e.m.f constant Kb

$$T_M \omega_m = e_b i_a$$

$$T_M = K_T i_a$$

$$e_b = K_b \omega_m$$

$$K_T i_a \omega_m = K_b \omega_m i_a$$

$$K_T = K_b$$