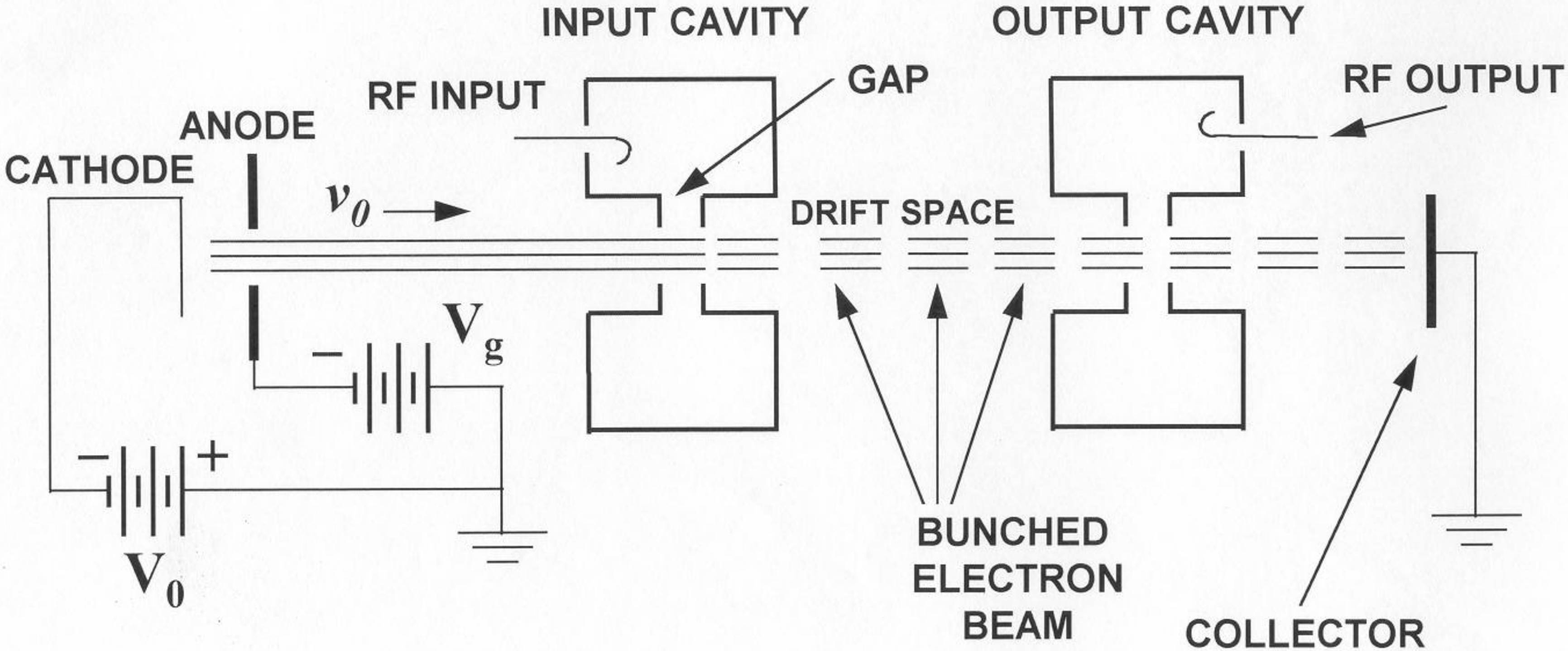
KLYSTRON STRUCTURE



General Characteristics

Electrons first accelerated by high dc voltage, V_0

$$v_0 = \sqrt{2eV_0/m} = 0.593\times10^6\sqrt{V_0} \ \text{(m/s)}$$

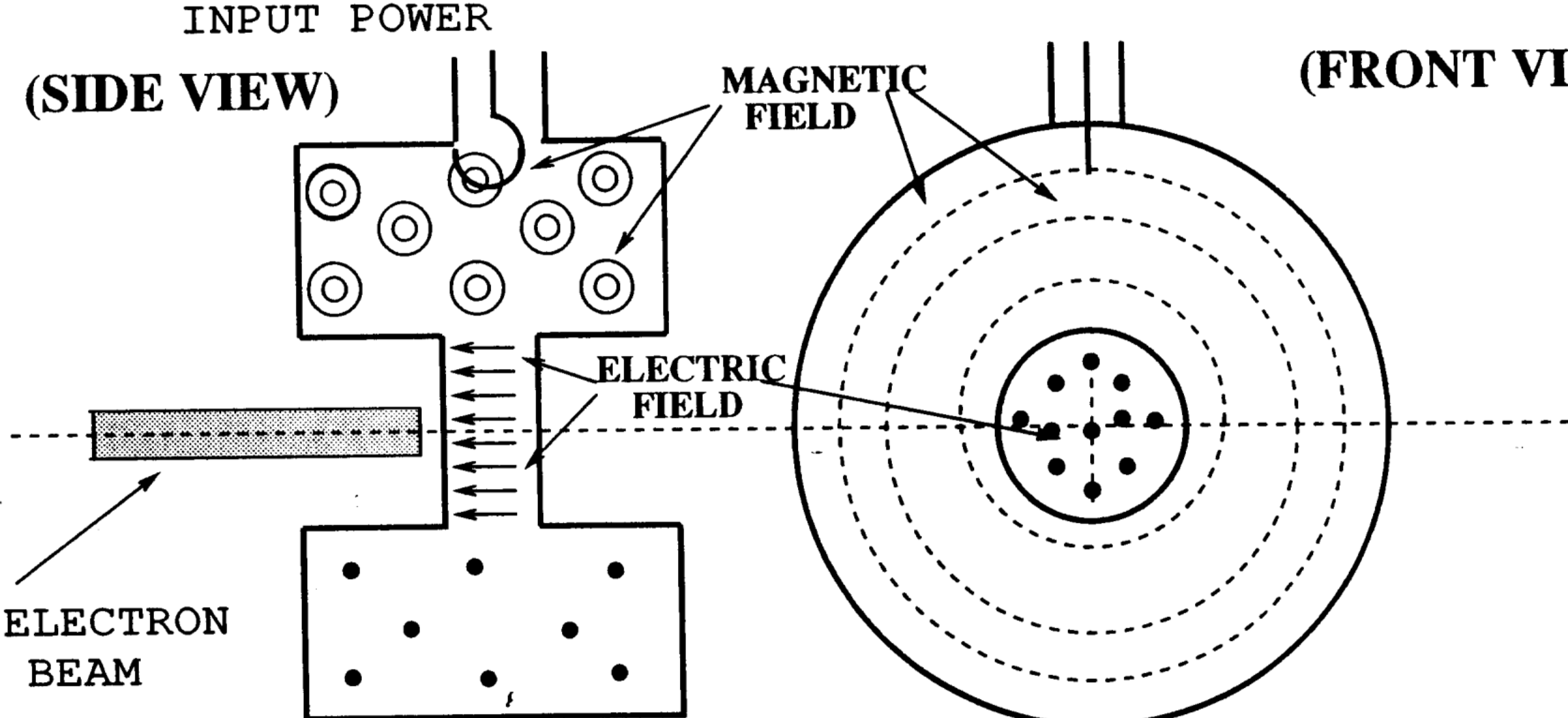
Microwave signal is applied to input terminals of buncher cavity. Gap voltage between the buncher grids with spacing, d:

$$V_s = V_1 \sin(\omega t), V_1 << V_0$$

(d) small therefore strong E fields for acceleration to modulate the streams of particles.

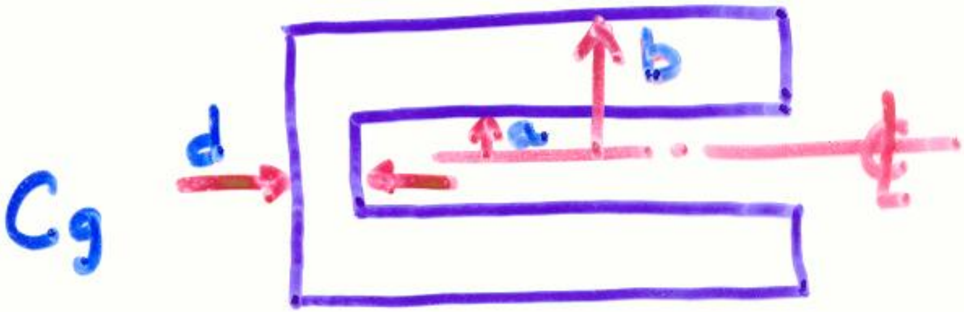
Also small d reduces transit time $d/v_0 << 1/f$ (period) at which the tube is to be operated (also the resonant frequency of the cavity).

KLYSTRON
TM-010 RESONATOR

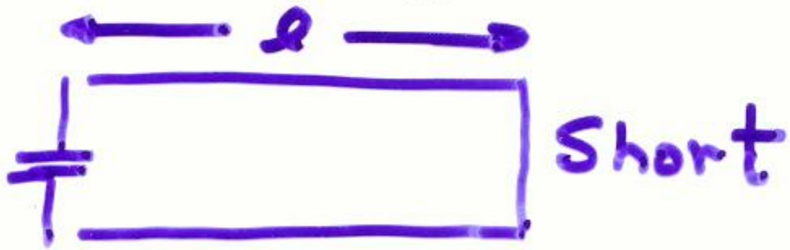


INPUT CAVITY
(BUNCHER)

Input Cavity



To find the resonant frequency, use transmission line equivalent to find



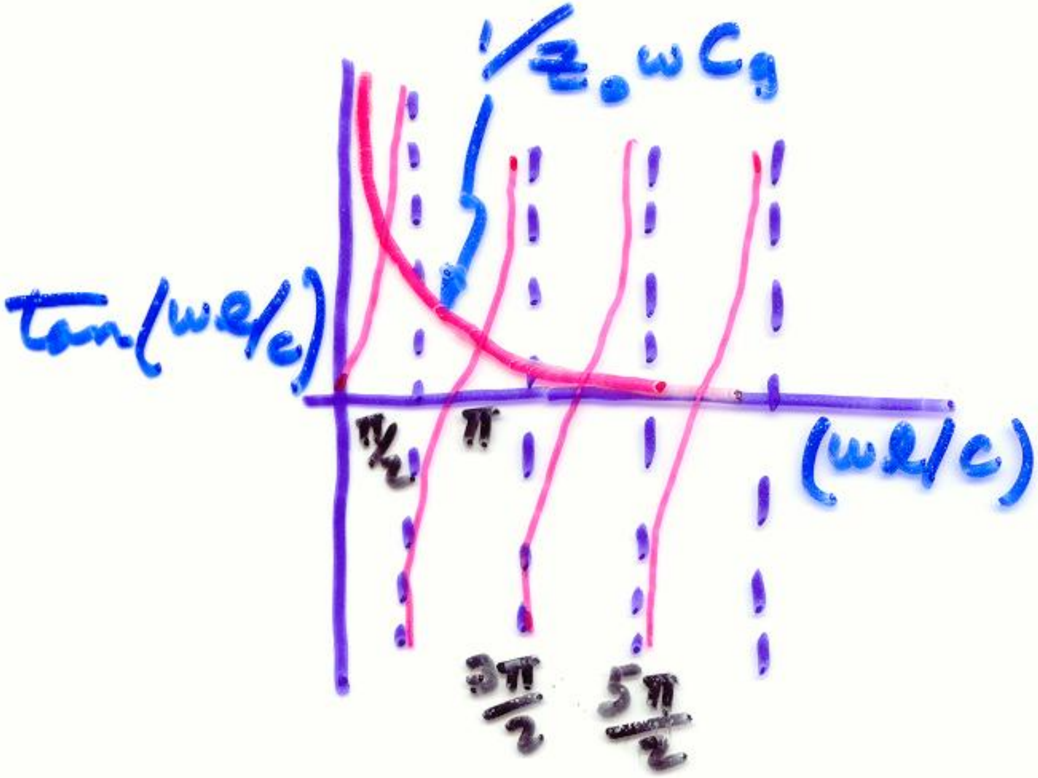
$$Z_i = Z_o \frac{Z_l \cos(\beta l) + jZ_o \sin(\beta l)}{Z_o \cos(\beta l) + Z_l \sin(\beta l)} = jZ_o \tan(\beta l) = \frac{1}{j\omega C_g}$$

from geometry: $C_g = \varepsilon_o \frac{\pi a^2}{d}$, d = gap spacing, neglecting fringing fields

Input Cavity

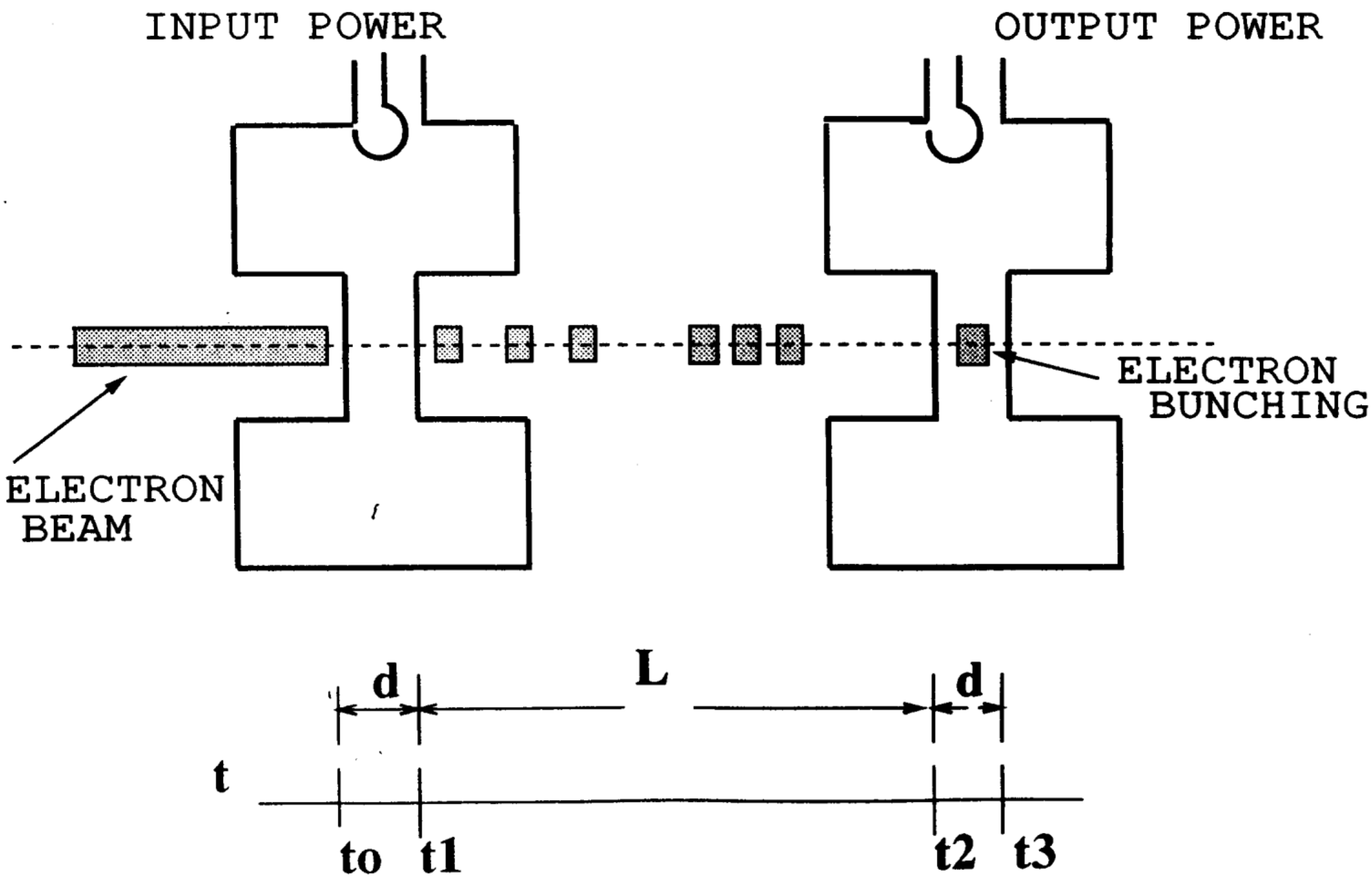
$$Z_o = \frac{60}{\sqrt{\varepsilon_r}} \ln(\tfrac{b}{a}) = 138 \log_{10}(\tfrac{b}{a}),\ \text{(TEM mode } Z_o \text{ for coaxial line)}$$

Hence, $\tan(\omega l/c) = \frac{1}{Z_o \omega C_g}$

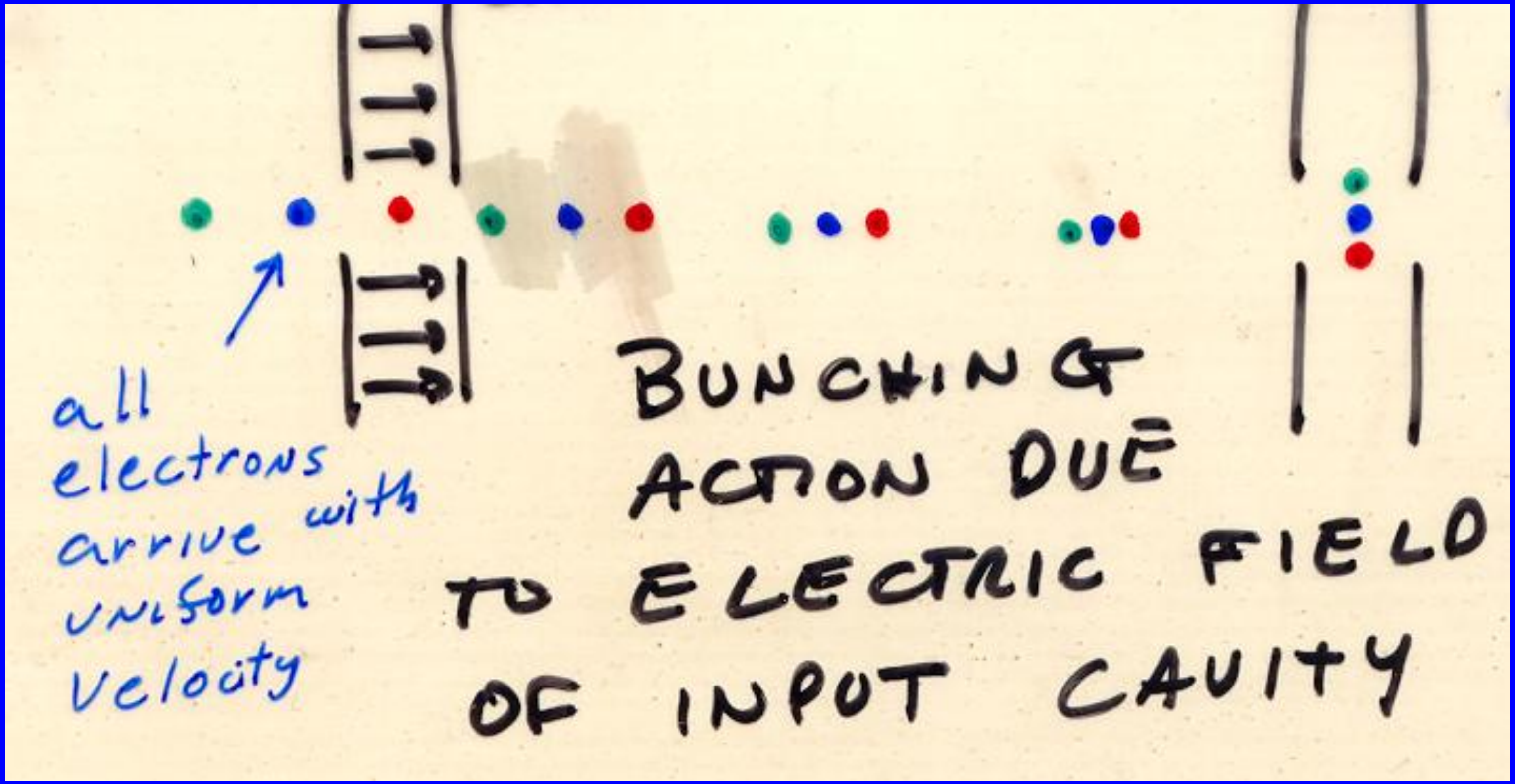


KLYSTRON

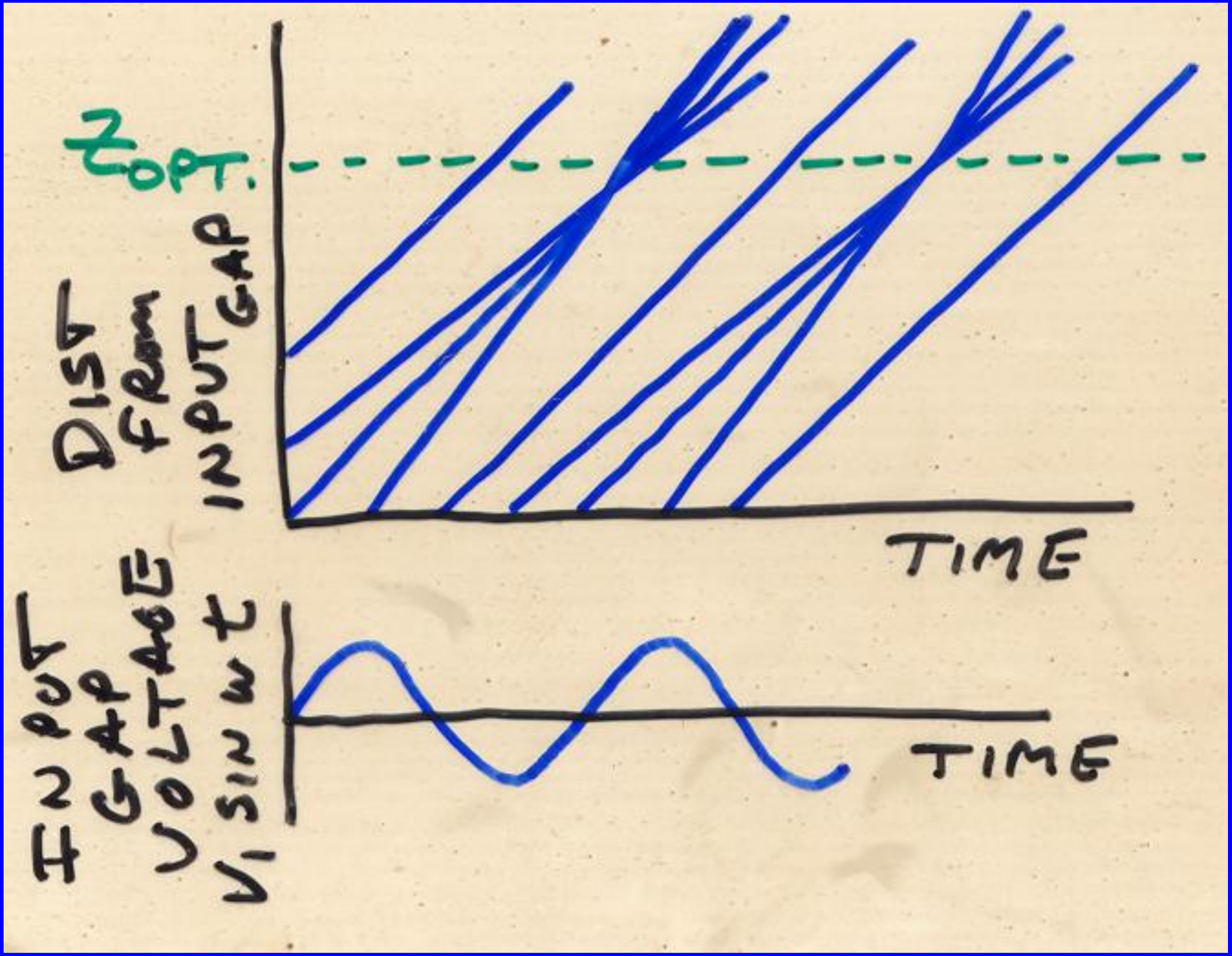
SIMPLIFIED SCHEMATIC



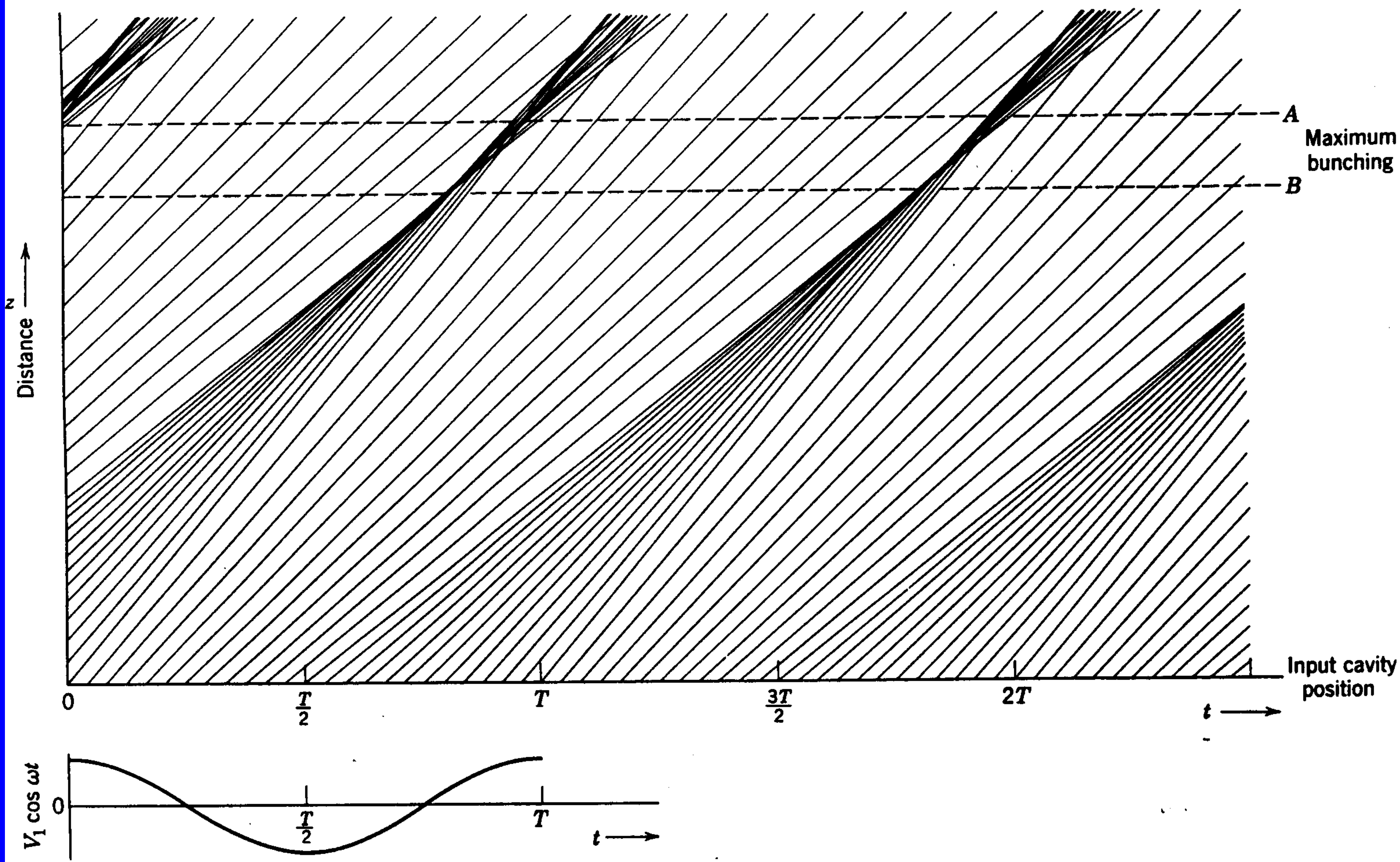
Bunching of Electrons



Time/Distance Applegate Diagram



APPLEGATE DIAGRAM SHOWING BUNCHING PROCESS



Beam coupling coefficient

$$\text{Force on electrons}: \vec{F} = m(d\vec{v}/dt) = eE \sin\omega t \ \hat{z}$$

$$\text{Integrate}: v = v_0 - (e/m)(E/\omega)\,(\cos\omega t_1 - \cos\omega t_0)$$

$$t_1 - t_0 = d/v_0 = \text{time across the gap}$$

$$v = v_0 - (e/m)(E/\omega)\{\cos(\omega t_0 + \omega d/v_0) - \cos\omega t_0\}$$

$$\text{let } A \equiv \omega t_0 + \omega d/(2v_0) \ \text{ and } B \equiv \omega d/(2v_0)$$

$$\therefore v = v_0 - (e/m)(E/\omega)\{\cos(A+B) - \cos(A-B)\}$$

$$\text{Note that the brackets } \{\,\} = -2 \sin A \,\text{Sin } B$$

Beam coupling coefficient

$$\therefore v = v_0 + 2(e/m)(E/\omega)\sin(\omega d/2v_0)\sin\omega(t_0 + d/2v_0)$$

$$= v_0 + (e/m)\,E\,(d/v_0)\,\beta\sin\omega(t_0 + d/2v_0)$$

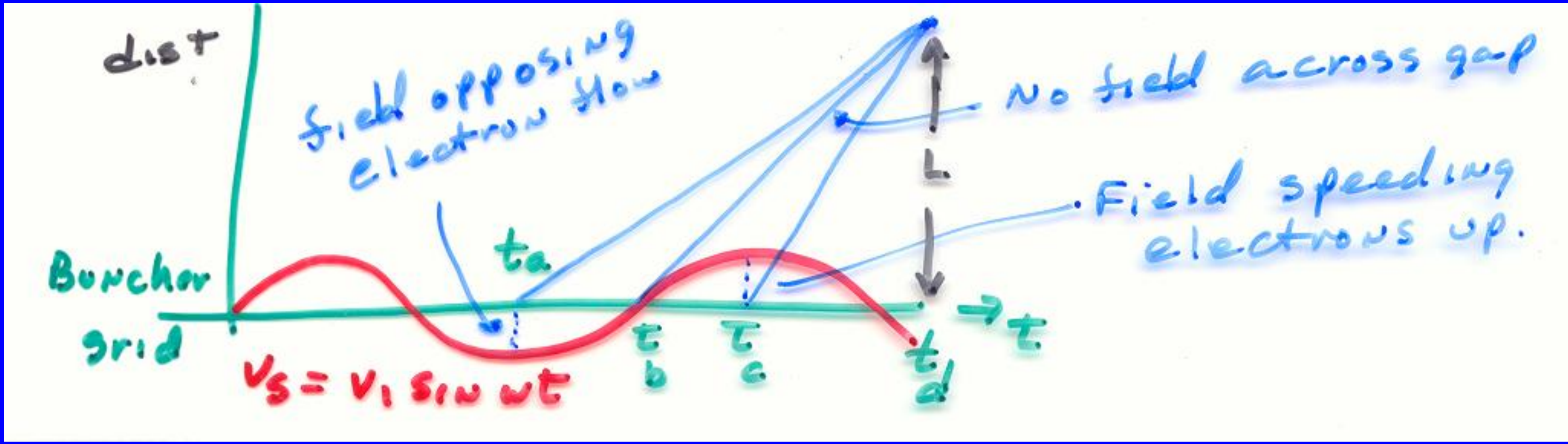
$$\text{where } \beta \equiv \frac{\sin(\omega d/2v_0)}{\omega d/2v_0} = \text{Beam coupling coefficient}$$

of the input cavity = ratio of ac current induced in exterior circuit to ac component of beam current

$\therefore$ electron velocity upon exit varies from electron to electron depending upon $(t_0 + d/2v_0)$, the average instant electron transit through the modulator gap.

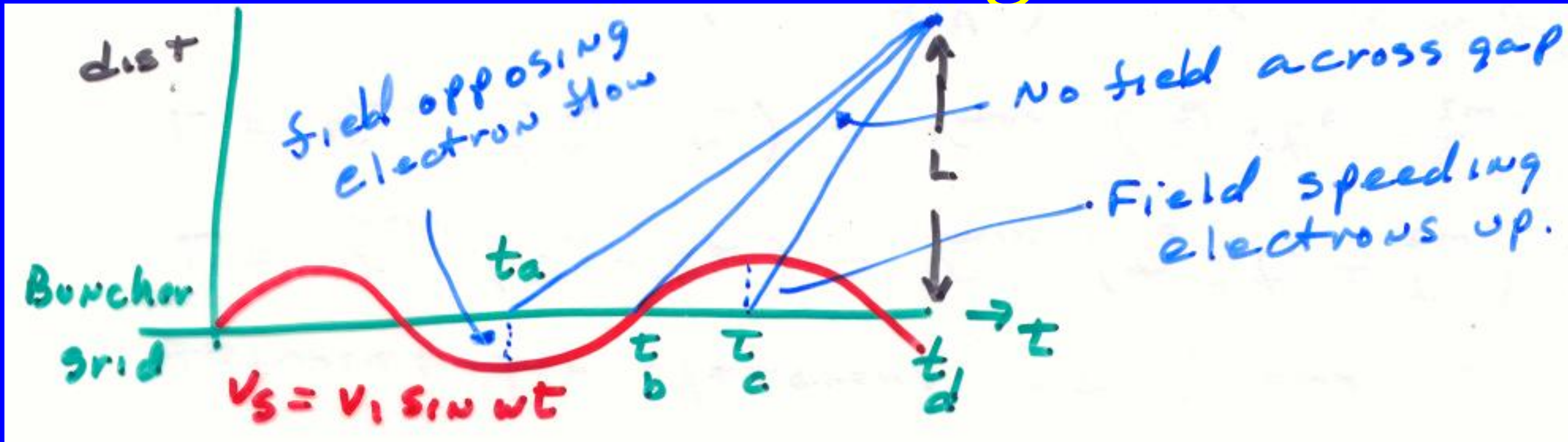
Electron Bunching Process

The net result of beam transit through the cavity is a sinusoidal
Beam velocity modulation at cavity frequency ω



Faster electrons "catch" up with slower electrons. At a certain
Distance L the electrons have "bunched" together. Here (at L)
A second cavity is placed in order to induce microwave fields
In the "output" of "catcher" cavity.

Electron Bunching Process



The distance from the buncher grid to the location of the of dense electron bunching for the electrons at t_b is $L = v_0 (t_d - t_b)$.

Distances for electrons at t_a and t_c are

$L = v_{min} (t_d - t_a) = v_{min} (t_d - t_b + \pi/(2\omega))$ (1)

$L = v_{max} (t_d - t_c) = v_{max} (t_d - t_b - \pi/(2\omega))$ (2), where

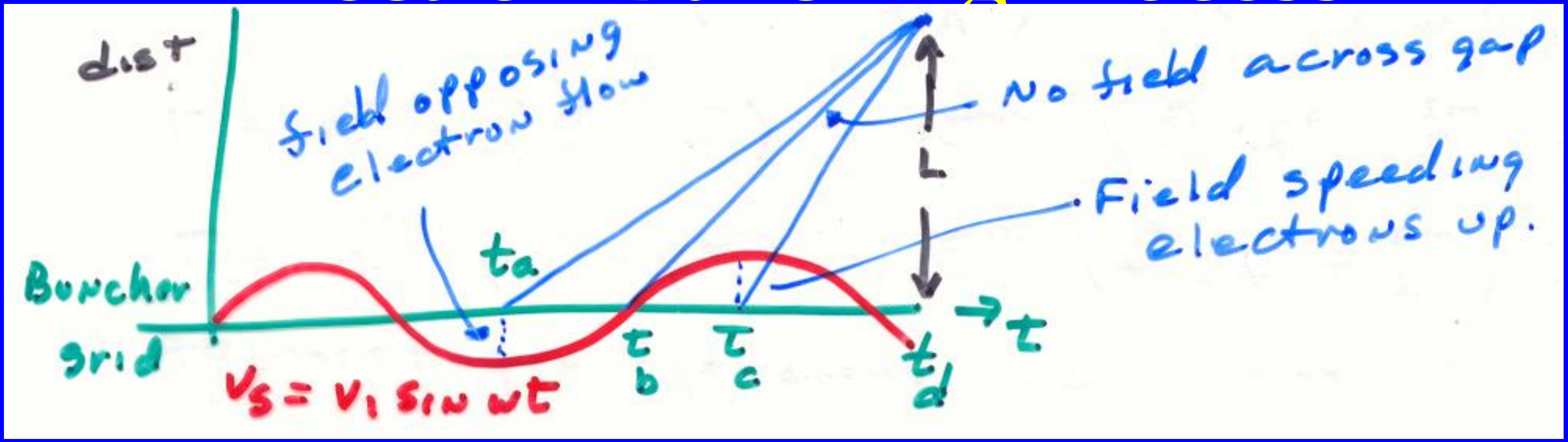
$v_{min} = v_0 \{1-(\beta V_1)/(2V_0)\}$; $V_0 = ½(m/e)(v_0^2)$, $V_1 = Ed$, and

$v_{max} = v_0 \{1+(\beta V_1)/(2V_0)\}$; equations (1) and (2) become

$L = v_0(t_d - t_b)+\{v_0 \pi/(2\omega)-v_0[(\beta V_1)/(2V_0)](t_d-t_b)-v_0[(\beta V_1)/(2V_0)]\pi/(2\omega)\}$

$L = v_0(t_d - t_b)+\{-v_0 \pi/(2\omega)+v_0[(\beta V_1)/(2V_0)](t_d-t_b)+v_0[(\beta V_1)/(2V_0)]\pi/(2\omega)\}$

Electron Bunching Process



For electrons at t_a, t_b, and t_c to meet at the same distance L means that terms in both brackets {} must = 0. therefore

$t_d - t_b = [(2V_0)/(v_0 \beta V_1)][v_0 \pi/(2\omega)][1-(\beta V_1)/(2V_0)] \sim \pi V_0/\omega\beta V_1$,

$L \sim v_0 \pi V_0/\omega\beta V_1$ (space charge neglected & not max degree of bunching)

Transit time in the field free region between grids is

$T = t_2 - t_1 = L/ v(t_1) = T_0 \{1 - [(\beta V_1)/(2V_0)] \sin [\omega t_1 - (\omega d)/(2v_0)]\}$

where $T_0=L/v_0$ and used $(1 + x)^{-1} \sim 1- x$; In radians

$\omega T = \omega t_2 - \omega t_1 = \omega L/ v_0 - X \sin [\omega t_1 - (\omega d)/(2v_0)]$, where

$X = (\omega L/ v_0) [(\beta V_1)/(2V_0)]$ = Bunching parameter of a Klystron

Electron Bunching Process

At the buncher gap a charge dQ_0 passing through at a time interval dt_0 is given by $dQ_0 = I_0\, dt_0 = i_2\, dt_2$, by conservation of charge, where i_2 = current at the catcher gap.

$t_2 = t_0 + \tau + T_0\ \{1 - [(\beta V_1)/(2V_0)]\ \sin\,[\omega t_0 + (\omega d)/(2v_0)]\}$

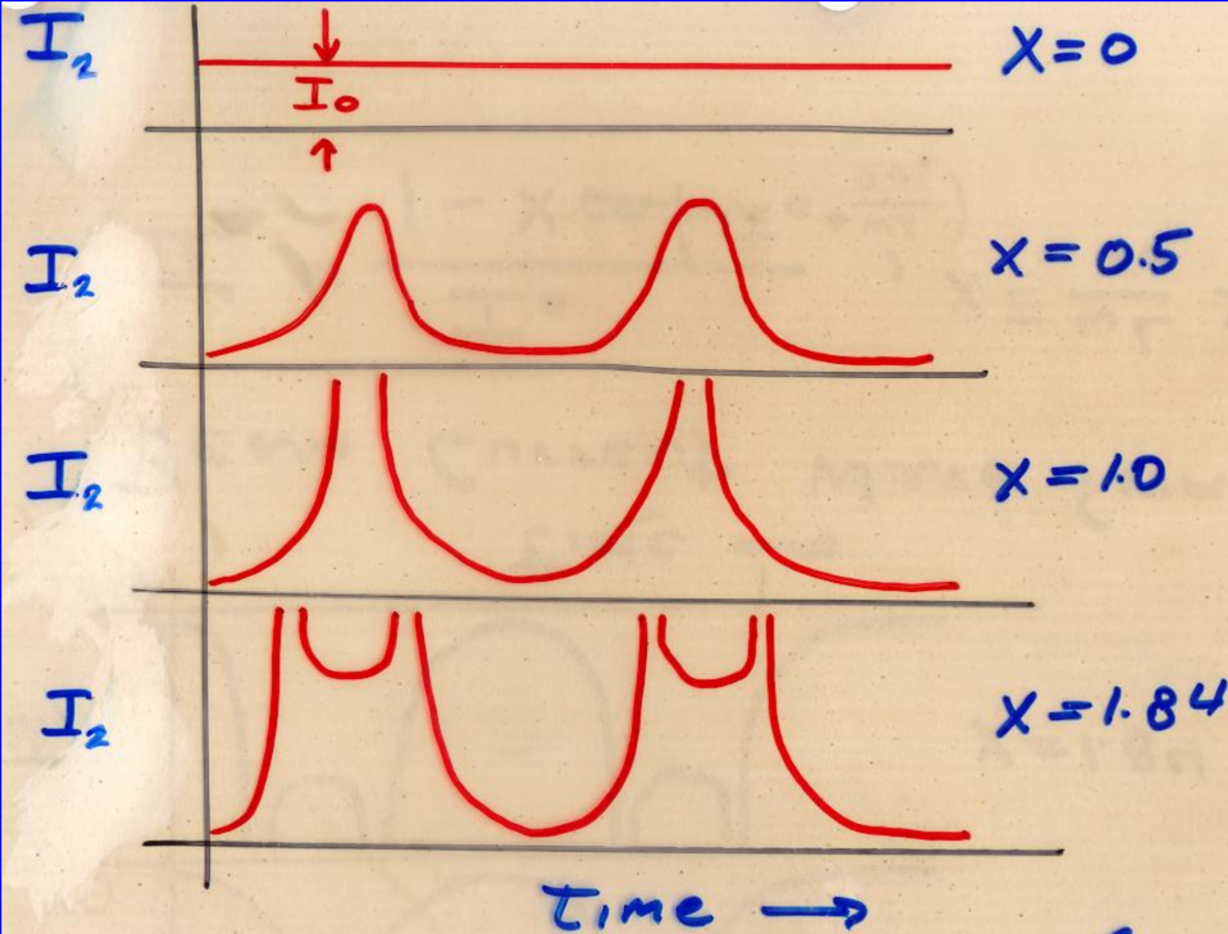
$dt_2\ /\ dt_0 = 1 - X \cos\,[\omega t_0 + (\omega d)/(2v_0)]$

$i_2\,(t_0) = I_0\ /\ \{1 - X \cos\,[\omega t_0 + (\omega d)/(2v_0)]\}$ = current arriving at catcher.

Using $t_2 = t_0 + \tau + T_0$,

$i_2\,(t_2) = I_0\ /\ \{1 - X \cos\,[\omega t_2 - (\omega L/v_0) - [(\omega d)/(2v_0)]\}$

Plot i_2 for various X (corresponding to different L providing β and $(V_1)/(2V_0\,)$ are fixed.)



Beam Current Wave forms

$$I_2 = \frac{I_0}{1 - X\cos\left(\omega\tau_0 + \frac{\omega d}{2v_0}\right)}, \quad X \equiv \frac{\omega L}{v_0}\,\frac{eEd}{mv_0^2}\,\beta$$

Electron Bunching Process

$\therefore$ Electron bunching corresponds to current peaks that take place and for $X \geq 1$; i_2 is rich in harmonics of the input frequency which is the resonant frequency of both cavities. (Klystron can be run as a harmonic generator).

Beam current at the catcher is a periodic waveform of period $2\pi/\omega$ about a dc current.

$\therefore$ expand i_2 in a Fourier Series:

$$i_2 = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t_2 + b_n \sin n\omega t_2)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} i_2 \, d(\omega t_2), \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} i_2 \cos n\omega t_2 \, d(\omega t_2)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} i_2 \sin n\omega t_2 \, d(\omega t_2)$$

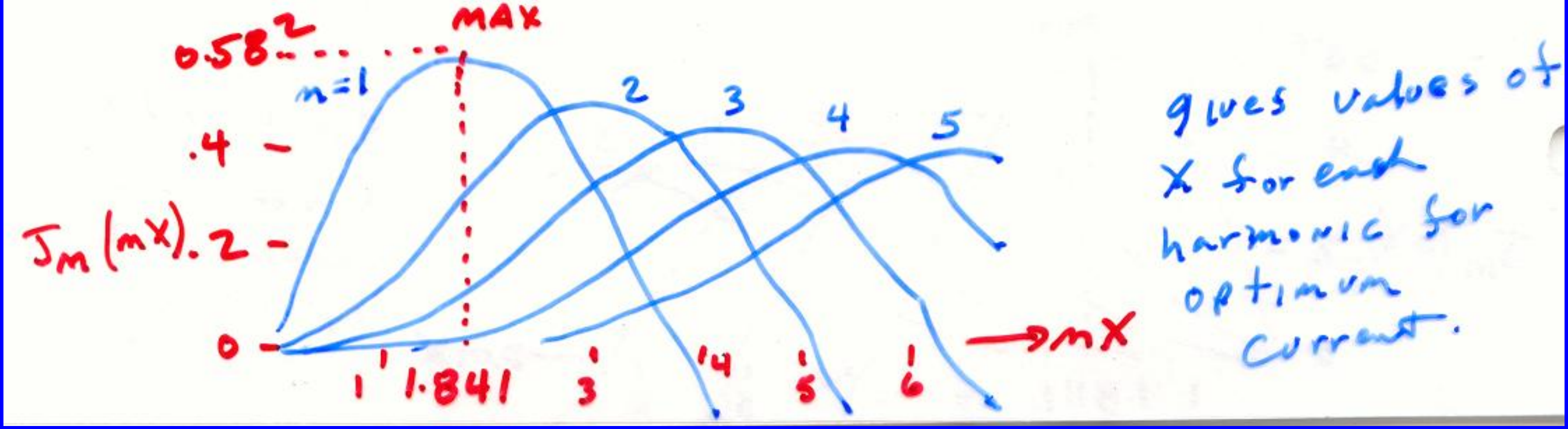
Electron Bunching Process

Insert i_2; $a_0 = I_0$

$$a_n = 2I_0 J_n(nX) \cos[n\omega d/(2v_0) + n\omega L/v_0]$$

$$b_n = 2I_0 J_n(nX) \sin[n\omega d/(2v_0) + n\omega L/v_0]$$

where $J_n(nX) = n^{th}$ order Bessel function of 1^{st} kind



Cavity Spacing

$$\therefore i_2 = I_0 + \sum_{n=1}^{\infty} 2\, I_0\, J_n(nX) \cos[n\omega(t_2 - \tau - T_0)]$$

Fundamental component of the beam current at the catcher cavity has a magnitude of $I_f = 2I_0 J_1(X)$.

Maximum amplitude when $X = 1.841$

$$\therefore\ X_{max} = 1.841 = \frac{\omega}{v_0} \frac{\beta V_1 L}{2V_0} \text{ or}$$

$$L_{opt} = \frac{3.682 v_0 V_0}{\omega \beta V_1} \approx 1.15\, L, \text{ from before.}$$

$L_{opt} \neq L$ due to the presence of harmonics in the beam.

Catcher Cavity

Phase of catcher gap voltage must be maintained in such a way that the bunched electrons as they pass through the grids encounter a retarding phase. Thus kinetic energy is transferred to the field of the catcher grid. The fundamental component of the induced current is given by:

$$i_{2\,induced} = \beta_0 i_2 = \beta_0\ 2\ I_0\ J_1(X)\ \cos[\omega(t_2 - \tau - T_0)]$$

β_0 = beam coupling coefficient of catcher gap.

If buncher and catcher cavities are identical then

$\beta_i = \beta_0$ and fundamental component of current induced in the catcher has a magnitude of

$$I_{2\,induced} = \beta_0 I_f = \beta_0\ 2\ I_0\ J_1(X)$$

Catcher Cavity- Output Power

R_{sho} = wall resistance of catcher cavity
R_B = beam loading resistance
R_L = external load resistance
R_{SH} = effective Shunt resistance
$I_2 = I_f$ = fundamental component of the beam current at the catcher cavity
V_2 = fundamental component of the catcher gap voltage
Output power delivered to the catcher and the load is given by

$$P_{out} = (1/2)\ (\beta_0 I_2)^2 R_{SH} = (1/2)\ \beta_0 I_2 V_2$$

$$\text{where } R_{SH} = R_{sho} // R_B // R_L$$

Efficiency and Mutual Conductance of Klystron

$$\text{Efficiency} = P_{out}/P_{in} = (1/2)\,\frac{\beta_0\, I_2\, V_2}{I_0\, V_0}.$$

$$\text{If coupling is perfect, } \beta_0 = 1 \text{ and } I_{2_{max}} = 2\, I_0\, (0.582)$$

$$\text{and } V_2 = V_0, \text{ then efficiency} = 58\%$$

$$\text{In practice efficiency} = 15 \text{ to } 30\%$$

$$\text{Mutual conductance, } G_m \equiv \frac{\text{induced output i}}{\text{input v}} = \frac{i_{2ind.}}{V_1}$$

$$= 2\,\beta_0\, I_0\, J_1(X)/V_1; \text{ use } V_1 = (2V_0/\beta_0)(v_0/\omega L)X$$

$$\frac{G_m}{G_0} = \beta_0^{\,2}\,\frac{\omega L}{v_0}\,\frac{J_1(X)}{X},\; G_0 \equiv \frac{I_0}{V_0}$$