RESONATOR'S Q-FACTORS

$$Q = \frac{2\pi \text{ ENERGY STORED}}{\text{ENERGY DISSIPATED PER CYCLE}}$$

UNLOADED Q: $Q_u = 2\pi f_o (L I^2/2)/(r I^2/2) = \omega_o L/r$

LOADED Q : $Q_L = \omega_o L/(r + Z_o) = Q_u/(1+ Z_o/r)$

COUPLING PARAMETER : $\beta = Z_o/r$; $Q_u = (1+ \beta) Q_L$

EXTERNAL Q : $Q_E = Q_u/\beta$; $1/Q_L = 1/Q_u + 1/Q_E$

LOADED Q: INCLUDES ALL DISSIPATION SOURCES

UNLOADED Q: INCLUDES ONLY INTERIOR DISSIPATION SOURCES TO CAVITY COUPLING SYSTEM

CIRCUIT PARAMETERS AND DEFINITIONS

$$Z_{in} = r + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z_{in} = r + j\hat{Z}_o\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)$$

where :

$$\hat{Z}_o = \sqrt{\frac{L}{C}}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

RESONATOR'S INPUT REFLECTION COEFFICIENT

$$\rho_{in} = \frac{Z_{in} - Z_o}{Z_{in} - Z_o} = \frac{r - Z_o + j\hat{Z}_o\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}{r + Z_o + j\hat{Z}_o\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)} = \frac{\left(\frac{r - Z_o}{\hat{Z}_o}\right) + j\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}{\left(\frac{r + Z_o}{\hat{Z}_o}\right) + j\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

DEFINITIONS AND RELATIONSHIPS AMONG THE RESONATOR'S Q'S

$$Q_u = \frac{\omega_o L}{r} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{r} = \sqrt{\frac{L}{C}} \cdot \frac{1}{r} = \frac{\hat{Z}_o}{r}$$

$$Q_L = \frac{\omega_o L}{r + Z_o} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{r + Z_o} = \sqrt{\frac{L}{C}} \cdot \frac{1}{r + Z_o} = \frac{\hat{Z}_o}{r + Z_o}$$

$$Q_E = \frac{\omega_o L}{Z_o} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{Z_o} = \sqrt{\frac{L}{C}} \cdot \frac{1}{Z_o} = \frac{\hat{Z}_o}{Z_o}$$

$$\frac{1}{Q_L} = \frac{1}{Q_E} + \frac{1}{Q_u}$$

AMPLITUDE MEASUREMENTS

The reflection coefficient is:

$$\rho_{in} = \frac{\left(\frac{1}{Q_u} - \frac{1}{Q_E}\right) + j\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}{\left(\frac{1}{Q_u} + \frac{1}{Q_E}\right) + j\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

Magnitude of the reflection coefficient is:

$$\left|\rho_{in}\right|^2 = \frac{\left(\frac{1}{Q_u} - \frac{1}{Q_E}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}{\left(\frac{1}{Q_u} + \frac{1}{Q_E}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}$$

Reflection Coefficient At Resonance :

$$\rho_o = \frac{\left(\frac{1}{Q_u} - \frac{1}{Q_E}\right)}{\left(\frac{1}{Q_u} + \frac{1}{Q_E}\right)}$$

At Angular Frequency $\omega = \omega_L$ Where:

$$\left(\frac{\omega_L}{\omega_o} - \frac{\omega_o}{\omega_L}\right)^2 = \frac{1}{Q_L^2} = \left(\frac{1}{Q_u} + \frac{1}{Q_E}\right)^2$$

The Reflection Coefficient is Given By:

$$\left|\rho_L\right|^2 = \frac{\left(\frac{1}{Q_u} - \frac{1}{Q_E}\right)^2 + \left(\frac{1}{Q_u} + \frac{1}{Q_E}\right)^2}{\left(\frac{1}{Q_u} + \frac{1}{Q_E}\right)^2 + \left(\frac{1}{Q_u} + \frac{1}{Q_E}\right)^2} = \frac{1}{2} + \frac{\left(\frac{1}{Q_u} - \frac{1}{Q_E}\right)^2}{2\left(\frac{1}{Q_u} + \frac{1}{Q_E}\right)^2} = \frac{1}{2} + \frac{1}{2}\left|\rho_o\right|^2$$

- MEASURE REFLECTION COEFFICIENT ρ_0 AT RESONANCE
- DETERMINE ρ_L FROM:

$$|\rho_L|^2 = \frac{1}{2} + \frac{1}{2}|\rho_o|^2$$

- OR USE CURVE OF ρ_L IN dB VS. ρ_o IN dB TO FIND ρ_L
- MEASURE THE FREQUENCIES FOR WHICH THE REFLECTION COEFFICIENT IS EQUAL TO ρ_L
- CALCULAT Q_L FROM :

$$Q_L = \frac{\omega_o \omega_L}{\omega_L^2 - \omega_o^2}$$

- CALCULATE Q_E FROM:

$$Q_E = \frac{2Q_L}{1 \pm |\rho_O|}$$

- THE SIGN TO USE IS DETERMINED FROM THE PHASE OF ρ_0 USE +VE SIGN FOR $r < Z_0$ AND -VE SIGN FOR $r < Z_0$