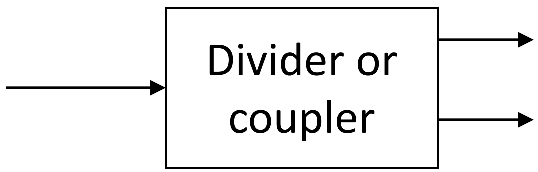
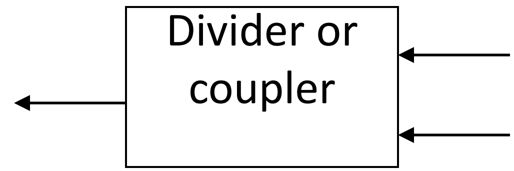
Power Dividers and Directional Couplers



Power division



Power combining

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \quad , \quad [S][S]^{*T} = \text{Identity matrix} \Rightarrow$$

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad , \quad |S_{12}|^2 + |S_{23}|^2 = 1 \quad , \quad |S_{13}|^2 + |S_{23}|^2 = 1$$

$$S_{13}^* S_{23} = 0 \quad , \quad S_{23}^* S_{12} = 0 \quad , \quad S_{12}^* S_{13} = 0$$

$\therefore$ At least two of the three parameters (S_{12}, S_{13}, S_{23}) must be zero.

A three port cannot be lossless, reciprocal, and matched at all ports.

Four-Port Network (Directional Couplers)

Assume all ports are matched

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$\text{Lossless} \Rightarrow \mathrm{S}_{13}^{*}S_{23}+S_{14}^{*}S_{24}=0 \;\;,\;\; \mathrm{S}_{14}^{*}S_{13}+S_{24}^{*}S_{23}=0$$

$$S_{14}^{*}\left(\left|S_{13}\right|^{2}-\left|S_{24}\right|^{2}\right)=0$$

$$\mathrm{S}_{12}^{*}S_{23}+S_{14}^{*}S_{34}=0 \;\;,\;\; \mathrm{S}_{14}^{*}S_{12}+S_{34}^{*}S_{23}=0 \Rightarrow$$

$$S_{23}\left(\left|S_{12}\right|^{2}-\left|S_{34}\right|^{2}\right)=0$$

$S_{14}=S_{23}$ which results in a directional coupler

$$\left|S_{12}\right|^2+\left|S_{13}\right|^2=1\ ,\ \left|S_{12}\right|^2+\left|S_{24}\right|^2=1$$

$$\left|S_{13}\right|^2+\left|S_{34}\right|^2=1\ ,\ \left|S_{24}\right|^2+\left|S_{34}\right|^2=1$$

$$\left|S_{13}\right|=\left|S_{24}\right|=\beta\ ,\ \left|S_{12}\right|=\left|S_{34}\right|=\alpha$$

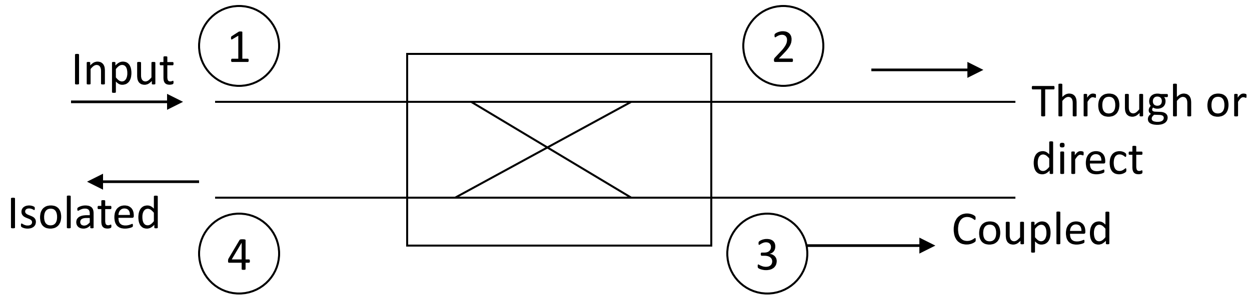
$$S_{12}=S_{34}=\alpha\ ,S_{13}=\beta e^{j\theta}\ ,S_{24}=\beta e^{j\phi}$$

$$S_{12}^*S_{13}+S_{24}^*S_{34}=0\Rightarrow\theta+\phi=\pi\pm2n\pi$$

$$\alpha^2+\beta^2=1$$

$$[S]=\begin{bmatrix}0&\alpha&j\beta&0\\\alpha&0&0&j\beta\\j\beta&0&0&\alpha\\0&j\beta&\alpha&0\end{bmatrix}\quad\text{(The symmetrical Coupler)}$$

$$[S]=\begin{bmatrix}0&\alpha&\beta&0\\\alpha&0&0&\beta\\\beta&0&0&\alpha\\0&\beta&\alpha&0\end{bmatrix}\quad\text{(The Antisymmetrical Coupler}$$



Directional Coupler

$$\text{Coupling} = \mathrm{C} = 10\log\frac{\mathrm{P}_1}{\mathrm{P}_3} = -20\log\beta \ \ \mathrm{dB}$$

$$\text{Directivity} = \mathrm{D} = 10\log\frac{\mathrm{P}_3}{\mathrm{P}_4} = 20\log\frac{\beta}{|\mathrm{S}_{14}|}\ \mathrm{dB}$$

$$\text{Isolation} = \mathrm{I} = 10\log\frac{\mathrm{P}_1}{\mathrm{P}_4} = -20\log|S_{14}| \ \ \mathrm{dB}$$

$$\mathrm{I} = \mathrm{D} + \mathrm{C} \ \ \mathrm{dB}$$

Hybrid couplers are special cases of Directional coupler, where the coupling factor is 3 dB.

$\alpha = \beta = \frac{1}{\sqrt{2}}$. The quadrature hybrid has a 90^0 phase shift between ports 2 and 3 when fed at port 1.

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

The magic T hybrid or rat - race has a 180^o phase differnce between ports 2 and 3 when fed at port 4.

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Directional Couplers

The coupling C is :

$$C = 10\ \log\ \frac{P_i}{P_f}$$

The directivity D is :

$$D = 10 \log \frac{P_f}{P_b}$$

For ideal coupler $S_{14} = S_{23} = 0$

$$S_{11} = S_{22} = 0$$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & S_{33} & S_{34} \\ 0 & S_{24} & S_{34} & S_{44} \end{bmatrix}$$

$$[S][S]^{*t} = U$$

$$S_{13}S_{33}^{*} = 0 \quad , \quad S_{24}S_{44}^{*} = 0$$

$$S_{33} = 0 = S_{44}$$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$S_{12}S_{24}^* + S_{13}S_{34}^* = 0 \qquad , \qquad S_{12}S_{13}^* + S_{24}S_{34}^* = 0$$

$$|S_{12}| \ \ |S_{24}| + |S_{13}|\,|S_{34}|$$

$$|S_{12}| \ \ |S_{13}| + |S_{24}|\,|S_{34}|$$

$$|S_{13}| = |S_{24}|$$

$$|S_{12}| = |S_{34}|$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

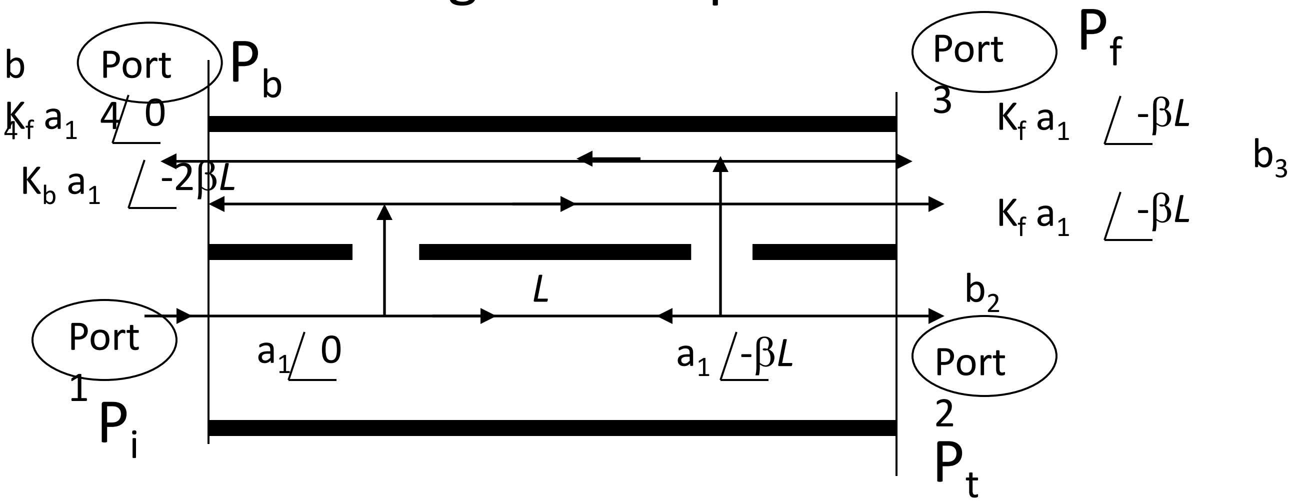
$$|S_{12}|^2 + |S_{24}|^2 = 1$$

$$S_{12} = C_1 \qquad , \qquad S_{13} = jC_2$$

$$[S] = \begin{bmatrix} 0 & C_1 & jC_2 & 0 \\ C_1 & 0 & 0 & jC_2 \\ jC_2 & 0 & 0 & C_1 \\ 0 & jC_2 & C_1 & 0 \end{bmatrix}$$

Directional Couplers

Two-hole Waveguide Couplers



K_f and K_r are the forward

and reverse aperture coupling coeffiecients

The coupling C is :

$$C = -20 \log 2|K_f|$$

The directivity D is :

$$D = 20\log \frac{2|K_f|}{|K_r||1+e^{-2j\beta L}|} = 20\log \frac{|K_f|}{|K_r||\cos \beta L|}$$

$$= 20\log \frac{|K_f|}{|K_r|} + 20\log|\sec \beta L|$$

The directivity is the sum of the directivity of the single aperture plus a directivity associated with the array.