WAVEGUIDE PROPERTIES

- **FOR A W/G FILLED WITH DIELECTRIC ε_r :**

$$\frac{1}{\lambda_1^2} = \frac{\varepsilon_r}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} \qquad \text{WHERE}$$

λ_1 IS WAVELENGTH IN DIELECTRIC

λ IS WAVELENGTH IN FREE SPACE

λ_g IS GUIDE WAVELENGTH

λ_C IS CUTOFF WAVELENGTH

WAVE IMPEDANCE Z_w IS:

$$Z_W = \frac{377}{\sqrt{\varepsilon_r}} \left(\lambda_g \big/ \lambda_1 \right) \qquad \text{FOR TE MODES}$$

$$= \frac{377}{\sqrt{\varepsilon_r}} \left(\lambda_1 / \lambda_g \right) \qquad \text{FOR TM MODES}$$

- **PROPAGATION PHASE CONSTANT:**

$$\beta = \frac{2\pi}{\lambda_g} \quad \text{RADIANS/UNIT LENGTH}$$

- **FOR RECTANGULAR GUIDE a X b, CUTOFF WAVELENGTH OF TE_{10} MODES ARE:**

$$\lambda_C = 2a \quad , \quad f_c = \frac{11.8}{\lambda_c}\sqrt{\varepsilon_r}$$

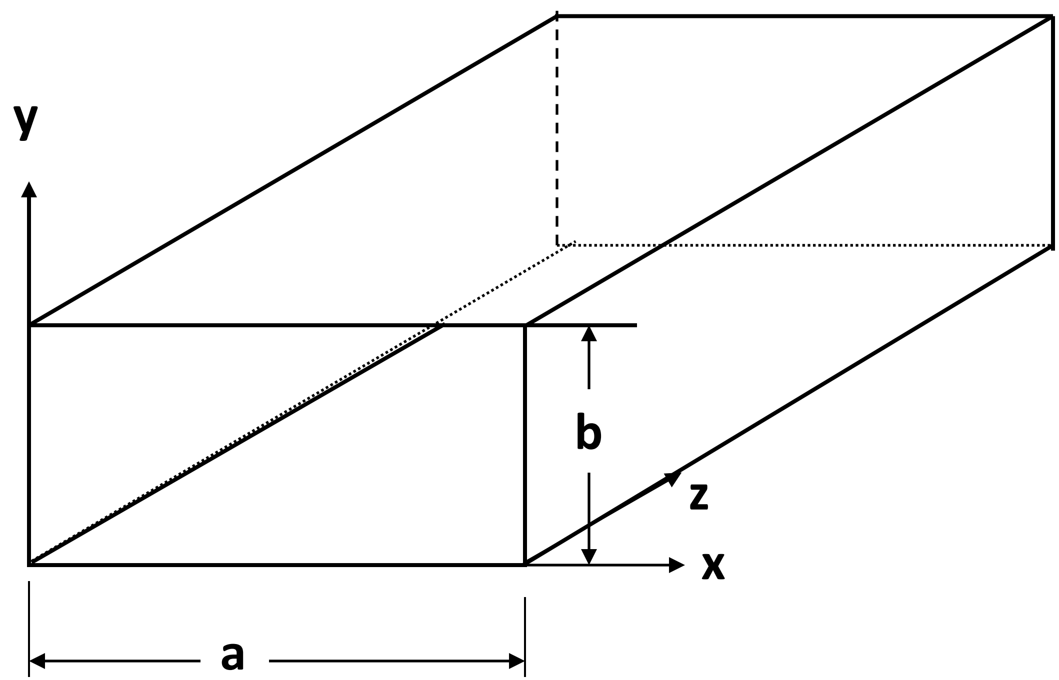
f_c **: CUT OFF FREQUENCY IN GHz (λ_c INCHES):**

- **FOR CIRCULAR WAVEGUIDE OF DIAMETER D CUTOFF WAVE LENGTH OF TE_{11} MODE IS:**

$\lambda_c = 1.706$ **D**

- **DOMINANT MODES ARE TE_{10} AND TE_{11} MODE FOR RECTANGULAR & CIRCULAR WAVEGUIDES**

RECTANGULAR WAVEGUIDE MODE FIELDS



CONFIGURATION

TE modes

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_c^2\right)h_z(x,y)=0$$

$$k_c^2 = k^2 - \beta^2$$

$$h_z(x,y) = X(x)Y(y)$$

$$\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}+k_c^2 = 0$$

$$\frac{1}{X}\frac{d^2X}{dx^2} = -k_x^2 \quad , \quad \frac{1}{Y}\frac{d^2Y}{dy^2} = -k_y^2$$

$$k_x^2 + k_y^2 = k_c^2$$

$$h_z(x,y) = (A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y)$$

$$H_z(x,y,z) = A_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$$

TE_{mn} MODES

$$H_z = \cos(\frac{m\pi x}{a})\ \cos(\frac{n\pi y}{b})\ e^{-j\beta z} \quad , \quad E_z = 0$$

$$E_x = Z_h H_y \quad , \quad E_y = -Z_h H_x$$

$$H_x = j\,\frac{\beta m\pi}{a k_c^2}\sin(\frac{m\pi x}{a})\ \cos(\frac{n\pi y}{b})\ e^{-j\beta z}$$

$$H_y = j\,\frac{\beta n\pi}{b k_c^2}\cos(\frac{m\pi}{a})\ \sin(\frac{n\pi y}{b})\ e^{-j\beta z}$$

$$Z_h = \frac{k\ Z_0}{\beta} \quad ; \quad k_c^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$$

$$\beta^2 = k^2 - k_c^2 \quad ; \quad \lambda_c = \frac{2ab}{(m^2b^2 + n^2a^2)^{1/2}}$$

$$f_{cmn} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2}$$

The dominant mode is TE_{10}

$$H_z = A_{10} \cos\frac{\pi x}{a} e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu a}{\pi} A_{10} \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$H_x = \frac{j\beta a}{\pi} A_{10} \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$E_x = E_z = H_y = 0$$

$$k_c = \pi / a \quad , \quad \beta = \sqrt{\mathrm{k}^2 - (\pi / a)^2}$$

$$P_{10} = \frac{1}{2}\mathrm{Re}\int_{x=0}^{a}\int_{y=0}^{b} \bar{E} \times \bar{H}^* \bullet \hat{z}\,dy\,dx = \frac{\omega\mu a^3 \left|A_{10}\right|^2 b}{4\pi^2}\mathrm{Re}(\beta)$$

$$P_\ell = \frac{R_s}{2}\int_C \left|\bar{J}_s\right|^2 d\ell = R_s \left|A_{10}\right|^2 (b + \frac{a}{2} + \frac{\beta^2 a^3}{2\pi^2})$$

$$\alpha_c = \frac{R_s}{a^3 b\beta k\eta}(2b\pi^2 + a^3 k^2) \quad \mathrm{Np/m}$$

TM_{mn} MODES

$$E_z = \sin(\frac{m\pi x}{a}) \ \sin(\frac{n\pi y}{b}) \ e^{-j\beta z}$$

$$H_z = 0$$

$$H_x = -E_y / Z_e \quad , \quad H_y = E_x / Z_e$$

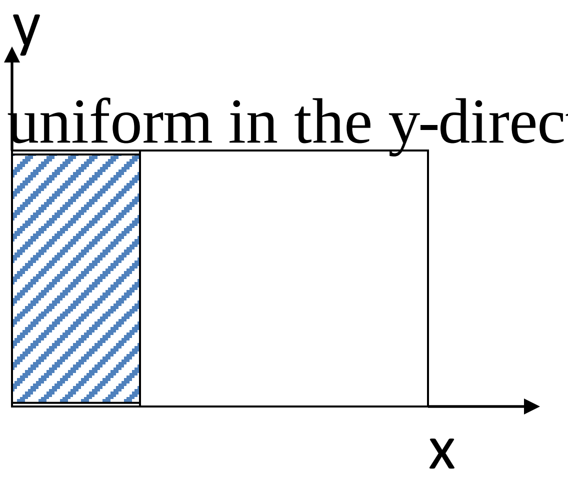
$$E_x = -j \frac{\beta m\pi}{a k_c^2} \cos(\frac{m\pi x}{a}) \ \sin(\frac{n\pi y}{b}) \ e^{-j\beta z}$$

$$E_y = -j \frac{\beta n\pi}{b k_c^2} \sin(\frac{m\pi}{a}) \quad \cos(\frac{n\pi y}{b}) \ e^{-j\beta z}$$

$$Z_e = \frac{\beta Z_0}{k} \quad ; \quad k_c^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$$

$$\beta^2 = k^2 - k_c^2 \quad ; \quad \lambda_c = \frac{2ab}{(m^2 b^2 + n^2 a^2)^{1/2}}$$

TE Modes of a Partially Loaded Waveguide



TE_{m0} have no y - variation and the structure is uniform in the y-direction

$$\left(\frac{\partial^2}{\partial x^2}+k_d^2\right)h_z=0 \quad \text{for } 0\le x\le t$$

$$\left(\frac{\partial^2}{\partial x^2}+k_a^2\right)h_z=0 \quad \text{for } t\le x\le a$$

k_d, k_a are the cutoff wavenumbers for dielectric and air regions

$$\beta=\sqrt{\varepsilon_r k_0^2-k_d^2}=\sqrt{k_0^2-k_a^2}$$

$$h_z=\begin{cases} A\cos k_d x+B\sin k_d x & \text{for } 0\le x\le t \\ C\cos k_a(a-x)+D\sin k_a(a-x) & \text{for } t\le x\le a \end{cases}$$

$$e_y = \begin{cases} \dfrac{j\omega\mu_0}{k_d}[-A\sin k_d x + B\cos k_d x] & \text{for } 0 \le x \le t \\ \dfrac{j\omega\mu_0}{k_a}[C\sin k_a(a-x) - D\cos k_a(a-x)) & \text{for } t \le x \le a \end{cases}$$

To satisfy the Boundary conditions that $E_y = 0$ at $x = 0$ and $x = a \Rightarrow$

$B = D = 0$, (E_y, H_x) are continuous at $x = t \Rightarrow$

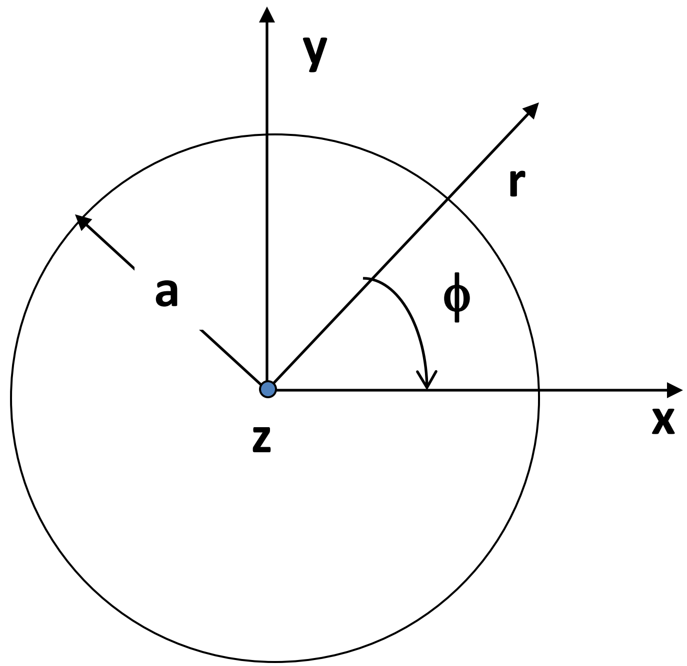
$$-\frac{A}{k_d}\sin k_d t = \frac{C}{k_a}\sin k_a(a-t)$$

$$A\cos k_d t = C\cos k_a(a-t)$$

$$k_a \tan k_d t + k_d \tan k_a(a-t) = 0$$

This is the characteristic equation that can yields to β

CIRCULAR WAVEGUIDE MODES



TE Modes

$$\nabla^2 H_z + k^2 H_z = 0$$

$$H_z(\rho,\phi,z) = h_z(\rho,\phi)e^{-j\beta z}$$

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + k_c^2\right)h_z(\rho,\phi) = 0$$

$$h_z(\rho,\phi) = R(\rho)\Phi(\phi)$$

$$\frac{1}{R}\frac{d^2R}{d\rho^2} + \frac{1}{\rho R}\frac{dR}{d\rho} + \frac{1}{\rho^2\Phi}\frac{d^2\Phi}{d\phi^2} + k_c^2 = 0$$

$$\frac{-1}{\Phi}\frac{d^2\Phi}{d\phi^2} = k_\phi^2 \quad , \quad \frac{d^2\Phi}{d\phi^2} + k_\phi^2\,\Phi = 0$$

$$\rho^2\frac{d^2R}{d\rho^2} + \rho\frac{dR}{d\rho} + (\rho^2 k_c^2 - k_\phi^2)R = 0$$

$$\Phi(\phi) = A\sin n\phi + B\cos n\phi \quad , \; k_\phi^2 = n^2$$

$$\rho^2 \frac{d^2R}{d\rho^2} + \rho\frac{dR}{d\rho} + (\rho^2 k_c^2 - n^2)R = 0$$

Bessel's Differential equation. The solution is :

$$\mathrm{R}(\rho) = \mathrm{C}J_n(k_c\rho) + DY_n(k_c\rho)$$

$J_n(k_c\rho), Y_n(k_c\rho)$ are the Bessel function of first and second kinds.

$Y_n(k_c\rho)$ is infinite at $\rho = 0 \Rightarrow \mathrm{D} = 0$

$$\mathrm{h_z}(\rho,\phi) = (A\sin n\phi + B\cos n\phi)\, J_n(k_c\rho)$$

The boundary condition $E_\phi(\rho,\phi) = 0$ at $\rho = \mathrm{a}$

$$\mathrm{E}_\phi(\rho,\phi,\mathrm{z}) = \frac{j\omega\mu}{k_c}(A\sin n\phi + B\cos n\phi)\, J_n'(k_c\rho)e^{-j\beta z}$$

$$J_n'(k_c a) = 0 \quad , \quad J_n'(p_{nm}') = 0 \quad p_{nm}' = \text{mth root of } J_n'$$

$$k_{cnm} = \frac{p_{nm}'}{a} \quad , \quad \beta_{\text{nm}} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - (\frac{p_{nm}'}{a})^2}$$

$$f_{cnm} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{p_{nm}'}{2\pi a\sqrt{\mu\varepsilon}}$$

$$E_\rho = \frac{-j\omega\mu n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$$

$$E_\phi = \frac{j\omega\mu}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}$$

$$H_\rho = \frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}$$

$$H_\phi = \frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$$

$$Z_{TE} = \frac{E_\rho}{H_\phi} = \frac{-E_\phi}{H_\rho} = \frac{\eta k}{\beta}$$

Dominant Mode is TE_{11}

$$H_z = A \sin\phi J_1(k_c\rho) e^{-j\beta z}$$

$$E_\rho = \frac{-j\omega\mu}{k_c^2 \rho} A \cos\phi J_1(k_c\rho) e^{-j\beta z}$$

$$E_\phi = \frac{j\omega\mu}{k_c} A \sin\phi J_1'(k_c\rho) e^{-j\beta z}$$

$$H_\rho = \frac{-j\beta}{k_c \rho} A \sin\phi J_1'(k_c\rho) e^{-j\beta z}$$

$$H_\phi = \frac{-j\beta}{k_c^2 \rho} A \cos\phi J_1(k_c\rho) e^{-j\beta z}$$

$$E_z = 0$$

TE_{nm} MODES

$$H_z = J_n\left(\frac{p'_{nm}\rho}{a}\right) \quad e^{-j\beta z} \qquad \begin{Bmatrix}\cos(n\varphi)\\ \sin(n\varphi)\end{Bmatrix}$$

$$E_z = 0$$

$$H_\rho = \frac{-j\beta p'_{nm} J'_n(p'_{nm}\rho / a)}{a k_c^2} e^{-j\beta z} \qquad \begin{Bmatrix}\cos(n\varphi)\\ \sin(n\varphi)\end{Bmatrix}$$

$$H_\varphi = \frac{-jn\beta J_n(p'_{nm}\rho / a)}{r k_c^2} e^{-j\beta z} \qquad \begin{Bmatrix}-\sin(n\varphi)\\ \cos(n\varphi)\end{Bmatrix}$$

$$E_\rho = Z_h H_\phi \quad ; \quad \mathrm{E}_\varphi = -Z_h H_\rho$$

p'_{nm} is the m'th zeros of $\mathrm{J}'_n(x)$

$$Z_h = kZ_0 / \beta \quad ; \quad \mathrm{k_c} = p'_{nm} / a$$

$$\beta^2 = k^2 - k_c^2 \quad ; \quad \lambda_c = 2\pi a / p'_{nm}$$

$\mathbf{TM_{nm}}$ MODES

$$E_z = J_n\left(\frac{p_{nm}\rho}{a}\right) \quad e^{-j\beta z} \quad \begin{Bmatrix} \cos(n\varphi) \\ \sin(n\varphi) \end{Bmatrix}$$

$$H_z = 0$$

$$E_\rho = \frac{-j\beta p_{nm} J_n'(p_{nm}'\rho / a)}{a k_c^2} e^{-j\beta z} \quad \begin{Bmatrix} \cos(n\varphi) \\ \sin(n\varphi) \end{Bmatrix}$$

$$E_\varphi = \frac{-jn\beta J_n(p_{nm}\rho / a)}{r k_c^2} e^{-j\beta z} \quad \begin{Bmatrix} -\sin(n\varphi) \\ \cos(n\varphi) \end{Bmatrix}$$

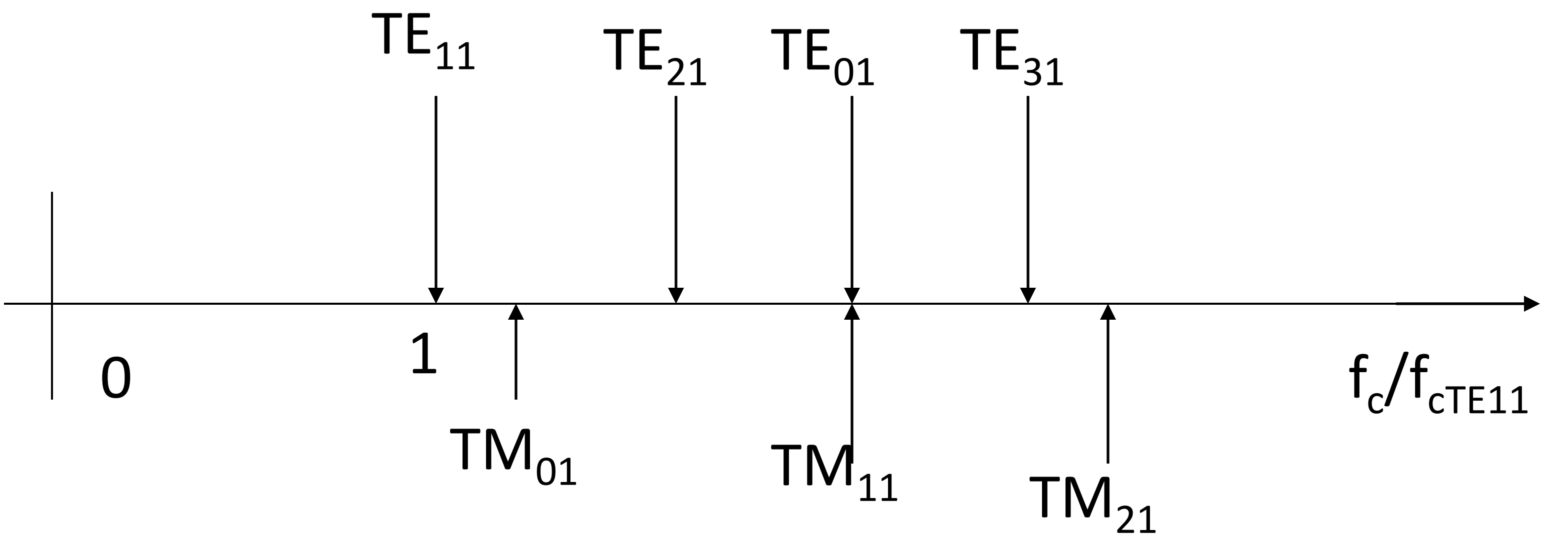
$$H_\rho = -E_\phi / Z_e \quad ; \quad \mathrm{H}_\varphi = E_\rho / Z_e$$

p_{nm} is the m'th zeros of $\mathrm{J}_n(x)$

$$Z_e = Z_0 \beta / k \quad ; \quad \mathrm{k_c} = p_{nm} / a$$

$$\beta^2 = k^2 - k_c^2 \quad ; \quad \lambda_c = 2\pi a / p_{nm}$$

Cutoff frequencies of the first few TE And TM modes in circular waveguide



ATTENUATION IN WAVEGUIDES

- **ATTENUATION OF THE DOMINANT MODES (TE_{m0}) IN A <u>COPPER</u> RECTANGULAR WAVEGUIDE DIM. *a* X *b*, AND (TE_{11}) CIRCULAR WAVEGUIDE, DIA. *D* ARE:**

$$\alpha_{c(TE_{m0})} = \frac{1.9x10^{-4}\sqrt{\varepsilon_r}\sqrt{f}}{b}\frac{\left[1+\frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right]}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \text{ dB/unit length}$$

$$\alpha_{c(TE_{11})} = \frac{3.8x10^{-4}\sqrt{\varepsilon_r}\sqrt{f}}{D}\frac{\left[\left(\frac{f_c}{f}\right)^2+0.42\right]}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \text{ dB/unit length}$$

WHERE *f* IS THE FREQUENCY IN GHz