

TM mode

- TM modes in rectangular wave guide
- TM₁₁
- TM₁₂
- TM₂₁

$$\lambda_{cm} = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}}$$

Propagation Of TE wave

$$\begin{aligned} E_x &= -\frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y} & E_x &= \frac{j\omega\mu}{h^2} \left(\frac{n\pi}{b}\right) C \cos\left(\frac{m\pi}{a}\right) x \sin\left(\frac{n\pi}{b}\right) y e^{(j\omega t - \gamma z)} \\ E_y &= -\frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x} & E_y &= -\frac{j\omega\mu}{h^2} \left(\frac{m\pi}{a}\right) C \sin\left(\frac{m\pi}{a}\right) x \cos\left(\frac{n\pi}{b}\right) y e^{(j\omega t - \gamma z)} \\ H_x &= -\frac{\gamma}{h^2} \frac{\partial H_z}{\partial x} & H_x &= \frac{j\beta}{h^2} \left(\frac{m\pi}{a}\right) C \sin\left(\frac{m\pi}{a}\right) x \cos\left(\frac{n\pi}{b}\right) y e^{(j\omega t - \gamma z)} \\ H_y &= -\frac{\gamma}{h^2} \frac{\partial H_z}{\partial y} & H_y &= \frac{j\beta}{h^2} \left(\frac{n\pi}{b}\right) C \cos\left(\frac{m\pi}{a}\right) x \sin\left(\frac{n\pi}{b}\right) y e^{(j\omega t - \gamma z)} \end{aligned}$$

- TE00
- TE01
- TE10
- TE11

- DOMINANT MODE : CUT OFF WAVELENGTH ASSUMES MAXIMUM VALUE
- $\lambda_{C01}=2b$
- $\lambda_{C10}=2a$
- $\lambda_{C11}=2ab/\sqrt{a^2+b^2}$
- $v_p=c/\sqrt{1-(\lambda_0/\lambda_c)^2}$

Wave impedance

- Ratio of electric field in one transverse direction to the strength of magnetic field in other transverse direction
- $Z_z = E_x/H_y = -E_y/H_x$

TM wave

$$Z_z = \frac{\sqrt{E_x^2 + E_y^2}}{\sqrt{H_x^2 + H_y^2}}$$

$$Z_{TM} = \frac{E_x}{H_y} = \frac{-\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2} \frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x}}$$

for TM wave $H_z = 0$, and $\gamma = j\beta$

$$Z_{TM} = \frac{E_x}{H_y} = \frac{\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x}}{-\frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x}} = \frac{\gamma}{j\omega\varepsilon} = \frac{j\beta}{j\omega\varepsilon} = \frac{\beta}{\omega\varepsilon}$$

$$\beta = \sqrt{\omega^2 \mu\varepsilon - \omega_c^2 \mu\varepsilon}$$

$$Z_{TM} = \frac{\sqrt{\omega^2 \mu\varepsilon - \omega_c^2 \mu\varepsilon}}{\omega\varepsilon}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

for air $\sqrt{\frac{\mu}{\varepsilon}} = 377\Omega = \eta$ (intrinsic impedance of free space)

$\lambda_0 < \lambda_c$, Z_{TM} is always less than free space impedance

TEwave

$$Z_z = \frac{\sqrt{E_x^2 + E_y^2}}{\sqrt{H_x^2 + H_y^2}}$$

$$Z_{TM} = \frac{E_x}{H_y} = \frac{-\gamma \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}}{-\gamma \frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x}}$$

$$= \frac{j\beta}{j\omega\varepsilon} = \sqrt{\frac{\mu\varepsilon}{\varepsilon_0\varepsilon}} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} = \eta \sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

for TM wave $E_z = 0$, and $\gamma = j\beta$

$$Z_{TE} = \frac{E_x}{H_y} = \frac{\frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2} \frac{\partial H_z}{\partial y}} = \frac{j\omega\mu}{\gamma} = \frac{\omega\mu}{\beta}$$

$$\beta = \sqrt{\omega^2 \mu\varepsilon - \omega_c^2 \mu\varepsilon}$$

$$Z_{TE} = \frac{\omega\mu}{\sqrt{\omega^2 \mu\varepsilon - \omega_c^2 \mu\varepsilon}} = \frac{\eta}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$= \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\eta}{\sqrt{1 - \left(\frac{\lambda_c}{\lambda_0}\right)^2}}$$

$\lambda_0 < \lambda_c$, Z_{TE} is always greater than free space impedance