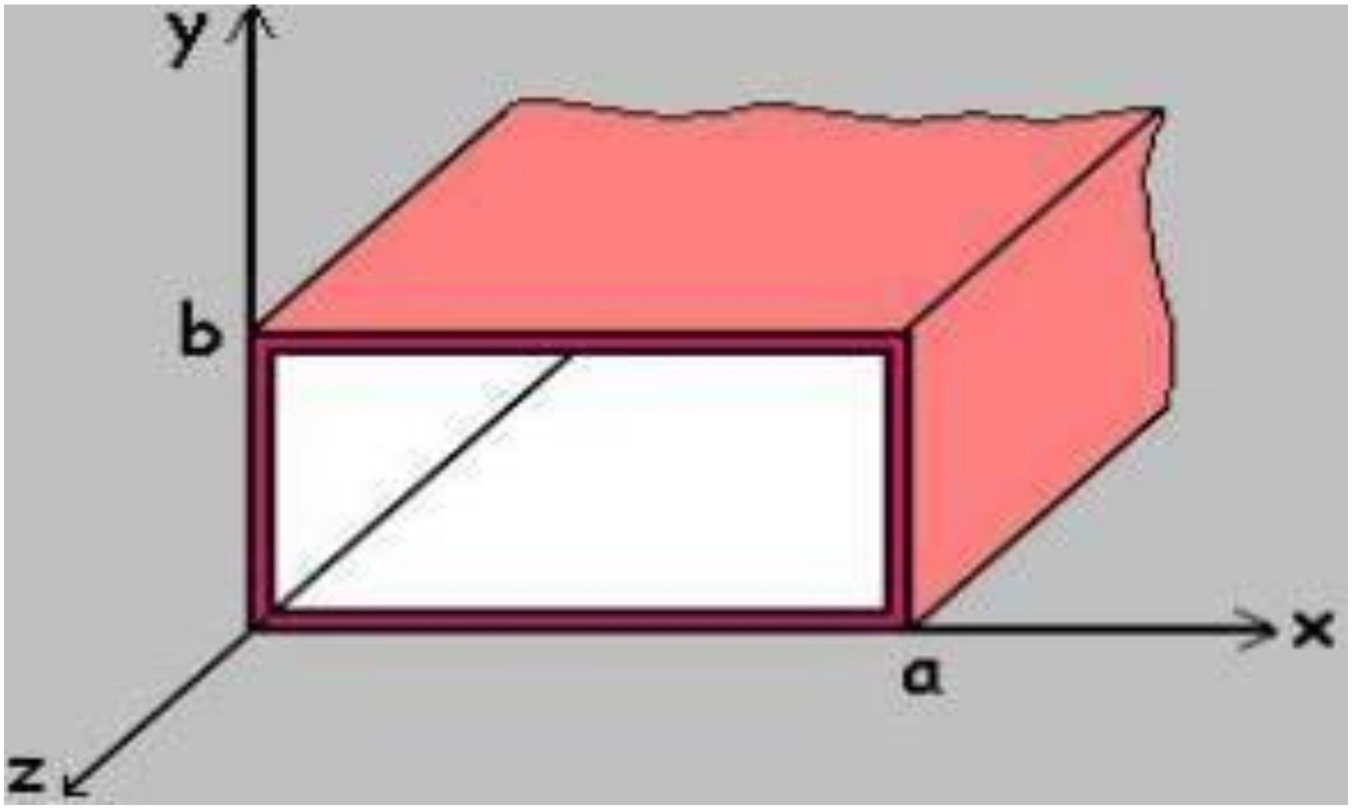
Lecture 5

Microwave



y
b
a
x
z

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) E_z = 0$$

$$\gamma^2 + \omega^2 \mu \varepsilon = h^2$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \quad \text{for TM wave}$$

similarly for T*E*wave

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0$$

$$where\, \omega^2 \mu \varepsilon + \gamma^2 = h^2$$

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$

$$E_y = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial y} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial x} - \frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x}$$

- Propagation of TEM mode
- TE and TM mode
- Dominant mode
- m and n

Propagation of TM wave in Rectangular wave guide

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0$$

assume '

$$E_z = XY$$

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 XY}{\partial x^2} = Y \frac{\partial^2 X}{\partial x^2}$$

$$\frac{\partial^2 E_z}{\partial y^2} = \frac{\partial^2 XY}{\partial y^2} = X \frac{\partial^2 Y}{\partial y^2}$$

$$Y\frac{\partial^2 X}{\partial x^2}+X\frac{\partial^2 Y}{\partial y^2}+h^2XY=0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}+h^2=0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}=-B^2$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}=-A^2$$

$$h^2=B^2+A^2$$

$$X=C_1\cos Bx+C_2\sin Bx$$

$$Y=C_3\cos Ay+C_4\sin Ay$$

BOUNDARY CONDITIONS

- First boundary condition

$$E_z = 0 \text{ at } y = 0 \text{ for all } x \to 0 \text{ to } a$$

- Second boundary Condition

$$E_Z = 0 \text{ at } x = 0 \text{for all } y \to 0 \text{ to } b$$

BOUNDARY CONDITIONS

- Third boundary condition

$$E_z = 0 \text{ at } y = b \text{ for all } x \to 0 \text{ to } a$$

- forth boundary Condition

$$E_Z = 0 \text{ at } x = a \text{ for all } y \to 0 \text{ to } b$$

$$X = C_1 \cos Bx + C_2 \sin Bx$$

$$Y = C_3 s \cos Ay + C_4 \sin Ay$$

$$E_z =$$

$$[C_1 \cos Bx + C_2 \sin Bx][C_3 s \cos Ay + C_4 \sin Ay]$$

$$wehaveEz = 0\ firstboundarycondition$$

$$[C_1 \cos Bx + C_2 \sin Bx][C_3 s \cos Ay + C_4 \sin Ay] = 0$$

$$[C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay] = 0$$

$$[C_1 \cos Bx + C_2 \sin Bx]C_3 = 0$$

$$[C_1 \cos Bx + C_2 \sin Bx] \neq 0, C_3 = 0$$

$$[C_1 \cos Bx + C_2 \sin Bx][C_4 \sin Ay] = 0$$

$$\sec ondboundaryconditon$$

$$C_1 C_4 \sin Ay = 0$$

$$C_4 \sin Ay \neq 0, C_1 = 0$$

$$[C_2 \sin Bx][C_4 \sin Ay] = E_z$$

$$third boundary condition$$

$$0 = [C_2 \sin Bx][C_4 \sin Ab] = E_z$$

$$\sin Ab = 0$$

$$A = \frac{n\pi}{b}$$

$$[C_2 \sin Bx][C_4 \sin Ay] = E_z$$

$$0 = [C_2 \sin Ba][C_4 \sin Ay] = E_z$$

$$\sin Ba = 0$$

$$B = \frac{m\pi}{a}$$

$$C_2 C_4 \sin Bx \sin Ay = E_z$$

$$C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y e^{-\gamma z} . e^{jwt}$$

$$E_z = C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y e^{j\omega - \gamma z}$$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$E_z = C\sin\left(\frac{m\pi}{a}\right)x\sin\left(\frac{n\pi}{b}\right)ye^{j\omega t-\gamma z}$$

$$E_x = \frac{-\gamma}{h^2}C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x\sin\left(\frac{n\pi}{b}\right)ye^{j\omega t-\gamma z}$$

$$E_z = E_o \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{j\omega t-\gamma z}$$

$$H_z = 0$$

- Other components are

$$E_x = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)C\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{j\omega t-\gamma z}$$

$$E_y = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)C\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{j\omega t-\gamma z}$$

$$H_x = -\frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right)C\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{j\omega t-\gamma z}$$

$$H_y = -\frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right)C\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{j\omega t-\gamma z}$$