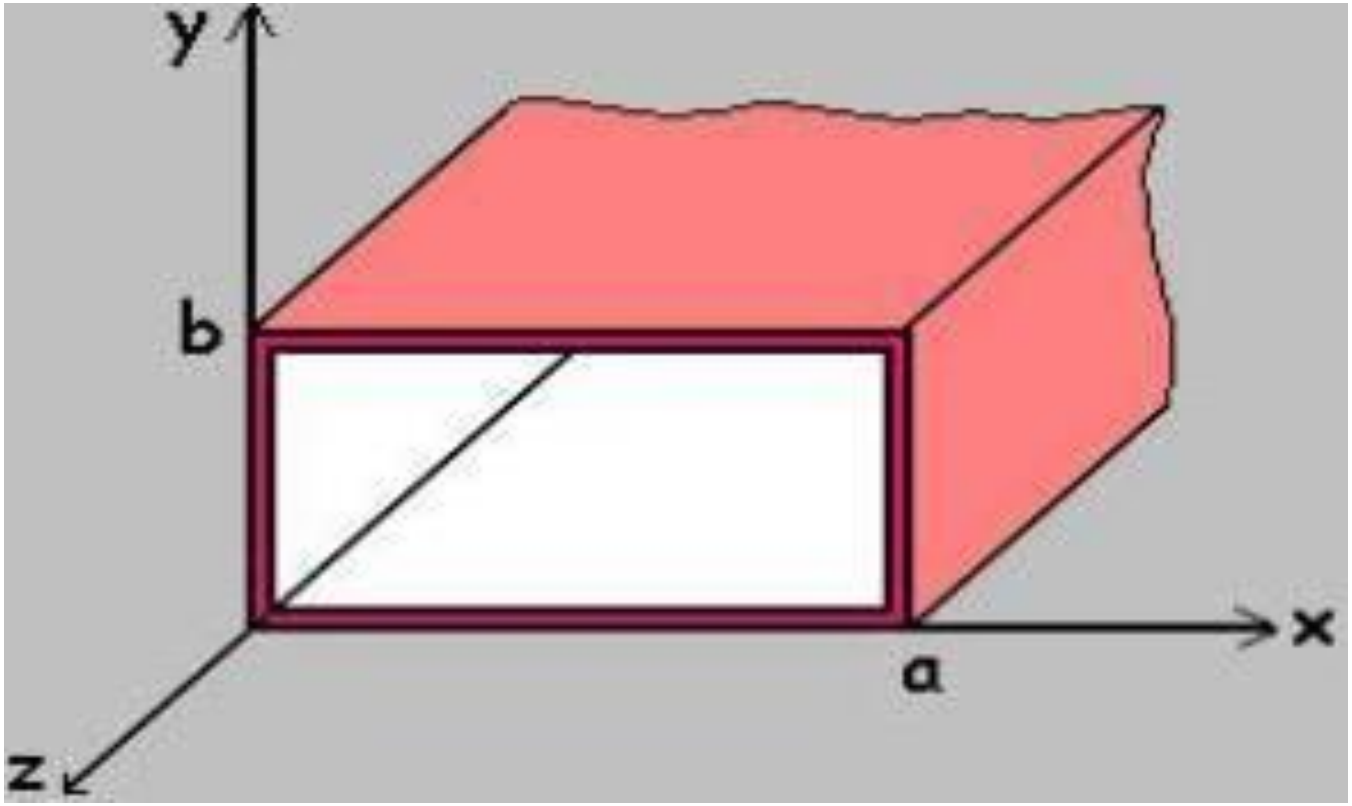
Lecture 4

Microwave


y
b
x
a
z

Propagation of waves in rectangular wave guide

- Wave equation for TE and TM wave

$$\nabla^2 H_z = -\omega^2 \mu\varepsilon H_z \, for TE wave (E_z = 0)$$

$$\nabla^2 E_z = -\omega^2 \mu\varepsilon E_z \, for TM wave (H_z = 0)$$

$$\exp anding \nabla^2 E_z in recangular coordinate system$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon E_z$$

$$\frac{\partial^2}{\partial z^2} = \gamma^2$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) E_z = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) E_z = 0$$

$$\gamma^2 + \omega^2 \mu \varepsilon = h^2$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \text{ for TM wave}$$

similarly for TE wave

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0$$

$$from Maxwell's first law$$

$$\nabla X H = j\omega\varepsilon E$$

$$\begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon[iE_x + jE_y + kE_z]$$

$$from Maxwell's first law$$

$$\nabla X H = j\omega\varepsilon E$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon[iE_x + jE_y + kE_z]$$

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$$

$$\frac{\partial H_z}{\partial y} + \gamma H_x = -j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

$$from Maxwell's second law$$

$$\nabla \times E = -j\omega\mu H$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu[iH_x + jH_y + kH_z]$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$H_y = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu}E_x$$

$$\frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma^2}{j\omega\mu}E_x = j\omega\varepsilon E_x$$

$$E_x\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right] = \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y}$$

$$E_x\left[-\omega^2\mu\varepsilon - \gamma^2\right] = \gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y}$$

$$E_x\left[-(\omega^2\mu\varepsilon + \gamma^2)\right] = \gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y}$$

$$where\, \omega^2 \mu\varepsilon + \gamma^2 = h^2$$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

- Propagation of TEM mode
- TE and TM mode
- Dominant mode
- m and n

Propagation of TM wave in Rectangular wave guide

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0$$

assume '

$$E_z = XY$$

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 XY}{\partial x^2} = Y \frac{\partial^2 X}{\partial x^2}$$

$$\frac{\partial^2 E_z}{\partial y^2} = \frac{\partial^2 XY}{\partial y^2} = X \frac{\partial^2 Y}{\partial y^2}$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -B^2$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -A^2$$

$$h^2 = B^2 + A^2$$

$$X = C_1 \cos Bx + C_2 \sin Bx$$

$$Y = C_3 \cos Ay + C_4 \sin Ay$$