Lecture 10
Standard Forms

STANDARD FORMS

SOP AND POS

- Boolean expressions can be manipulated into many forms.
- Some standardized forms are required for Boolean expressions to simplify communication of the expressions.
  - **Sum-of-products (SOP)**
    - Example:

$$F(A, B, C, D) = AB + \overline{B}C\overline{D} + AD$$

  - **Products-of-sums (POS)**
    - Example:

$$F(A, B, C, D) = (A + B)(\overline{B} + C + \overline{D})(A + D)$$

Minterms and Maxterms

Minterms and Maxterms

- MINTERMS AND MAXTERMS:

n binary variables can be combined to form 2^n terms (AND terms), called *minterms* or standard products.

In a similar fashion, *n* binary variables can be combined to form 2^n terms (OR terms), called *maxterms* or standard sums.

* Note that each maxterm is the complement of its corresponding minterm and vice versa.

Minterms and Maxterms (continued)

Minterms and Maxterms for Three Binary Variables

x y z	**Minterms**		**Maxterms**	
0 0 0	$x'y'z'$	m_o	$x+y+z$	M_o
0 0 1	$x'y'z$	m_1	$x+y+z'$	M_1
0 1 0	$x'yz'$	m_2	$x+y'+z$	M_2
0 1 1	$x'yz$	m_3	$x+y'+z'$	M_3
1 0 0	$xy'z'$	m_4	$x'+y+z$	M_4
1 0 1	$xy'z$	m_5	$x'+y+z'$	M_5
1 1 0	xyz'	m_6	$x'+y'+z$	M_6
1 1 1	xyz	m_7	$x'+y'+z'$	M_7

STANDARD FORMS

MINTERMS

The following table gives the minterms for a **three-input** system

A	B	C	m_0 $\overline{A}\,\overline{B}\,\overline{C}$	m_1 $\overline{A}\,\overline{B}C$	m_2 $\overline{A}B\overline{C}$	m_3 $\overline{A}BC$	m_4 $A\overline{B}\,\overline{C}$	m_5 $A\overline{B}C$	m_6 $AB\overline{C}$	m_7 ABC
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

STANDARD FORMS

SUM OF MINTERMS

- **Sum-of-minterms** standard form expresses the Boolean or switching expression in the form of a **sum of products** using **minterms**.
  - For instance, the following Boolean expression using minterms

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C$$

could instead be expressed as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = m_0 + m_1 + m_4 + m_5$$

or more compactly

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \sum m(0, 1, 4, 5) = \textit{one-set}(0, 1, 4, 5)$$

STANDARD FORMS

MAXTERMS

- The following table gives the maxterms for a **three-input** system

A	B	C	M_0 $A+B+C$	M_1 $A+B+\overline{C}$	M_2 $A+\overline{B}+C$	M_3 $A+\overline{B}+\overline{C}$	M_4 $\overline{A}+B+C$	M_5 $\overline{A}+B+\overline{C}$	M_6 $\overline{A}+\overline{B}+C$	M_7 $\overline{A}+\overline{B}+\overline{C}$
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

STANDARD FORMS

PRODUCT OF MAXTERMS

- **Product-of-maxterms** standard form expresses the Boolean or switching expression in the form of **product of sums** using **maxterms**.
  - For instance, the following Boolean expression using maxterms

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = (\mathbf{A} + \mathbf{B} + \overline{\mathbf{C}})(\overline{\mathbf{A}} + \mathbf{B} + \mathbf{C})(\overline{\mathbf{A}} + \overline{\mathbf{B}} + \overline{\mathbf{C}})$$

could instead be expressed as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = M_1 \cdot M_4 \cdot M_7$$

or more compactly as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \prod M(1, 4, 7) = \textit{zero-set}(1, 4, 7)$$

STANDARD FORMS

MINTERM AND MAXTERM EXP.

Given an arbitrary Boolean function, such as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \mathbf{AB} + \overline{\mathbf{B}}(\overline{\mathbf{A}} + \overline{\mathbf{C}})$$

how do we form the canonical form for:

- **sum-of-minterms**
  - Expand the Boolean function into a sum of products. Then take each term with a missing variable **X** and **AND** it with $\mathbf{X} + \overline{\mathbf{X}}$.
- **product-of-maxterms**
  - Expand the Boolean function into a product of sums. Then take each factor with a missing variable **X** and **OR** it with $\mathbf{X}\overline{\mathbf{X}}$.

STANDARD FORMS

FORMING SUM OF MINTERMS

- Example

$$F(A, B, C) = AB + \overline{B}(\overline{A} + \overline{C}) = AB + \overline{A}\overline{B} + \overline{B}\overline{C}$$

$$= AB(C + \overline{C}) + \overline{A}\overline{B}(C + \overline{C}) + (A + \overline{A})\overline{B}\overline{C}$$

$$= \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + A\overline{B}\overline{C} + AB\overline{C} + ABC$$

$$= \sum m(0, 1, 4, 6, 7)$$

A	B	C	F	
0	0	0	1	← 0
0	0	1	1	← 1
0	1	0	0	
0	1	1	0	
1	0	0	1	← 4
1	0	1	0	
1	1	0	1	← 6
1	1	1	1	← 7

Minterms listed as 1s in Truth Table

STANDARD FORMS

FORMING PROD OF MAXTERMS

- Example

$$F(A, B, C) = AB + \overline{B}(\overline{A} + \overline{C}) = AB + \overline{A}\overline{B} + \overline{B}\overline{C}$$

$$= (A + \overline{B})(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})$$ (using distributivity)

$$= (A + \overline{B} + C\overline{C})(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})$$

$$= (A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})$$

$$= \prod M(2, 3, 5)$$

A	B	C	F	
0	0	0	1	
0	0	1	1	
0	1	0	0	← 2
0	1	1	0	← 3
1	0	0	1	
1	0	1	0	← 5
1	1	0	1	
1	1	1	1	

Maxterms listed as 0s in Truth Table

STANDARD FORMS

CONVERTING MIN AND MAX

- Converting between sum-of-minterms and product-of-maxterms
  - The two are complementary, as seen by the truth tables.
  - To convert interchange the $\sum$ and $\prod$, then use missing terms.
    - Example: The example from the previous slides

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \sum m(0, 1, 4, 6, 7)$$

is re-expressed as

$$\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \prod M(2, 3, 5)$$

where the numbers 2, 3, and 5 were missing from the minterm representation.

Σminterms and Πmaxterms

- Given the truth table, express F_1 in sum of minterms

x	y	z	F_1	F_2
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$F_1(x,y,z) = \sum(1,4,5,6,7) = m_1 + m_4 + m_5 + m_6 + m_7$$

$$= (x'y'z) + (xy'z') + (xy'z) + (xyz') + (xyz)$$

- Find F_2

Σminterms and Πmaxterms

- Repeat for product of maxterms.

x	*y*	*z*	F_1	F_2
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$F_1(x,y,z) = \prod(0,2,3) = M_0 \cdot M_2 \cdot M_3$$

$$= (x+y+z)(x+y'+z)(x+y'+z')$$

Σminterms and Πmaxterms

Express the Boolean function $F = x + y'z$ in a sum of minterms, and then in a product of Maxterms.

$$x = x(y + y') = xy + xy'$$

$$xy = xy(z + z') = xyz + xyz'$$

$$xy' = xy'(z + z') = xy'z + xy'z'$$

$$y'z = y'z(x + x') = xy'z + x'y'z$$

Adding all terms and excluding recurring terms:

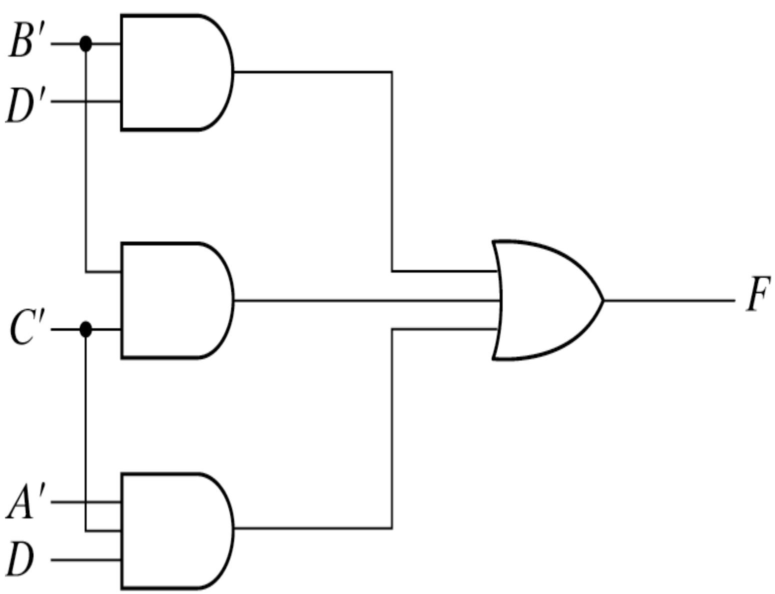
$$F(x, y, z) = x'y'z + xy'z' + xy'z + xyz' + xyz$$ **(SOP)**

$$F(x, y, z) = m_1 + m_4 + m_5 + m_6 + m_7 = \sum(1,4,5,6,7)$$

Product of maxterms (POS)?

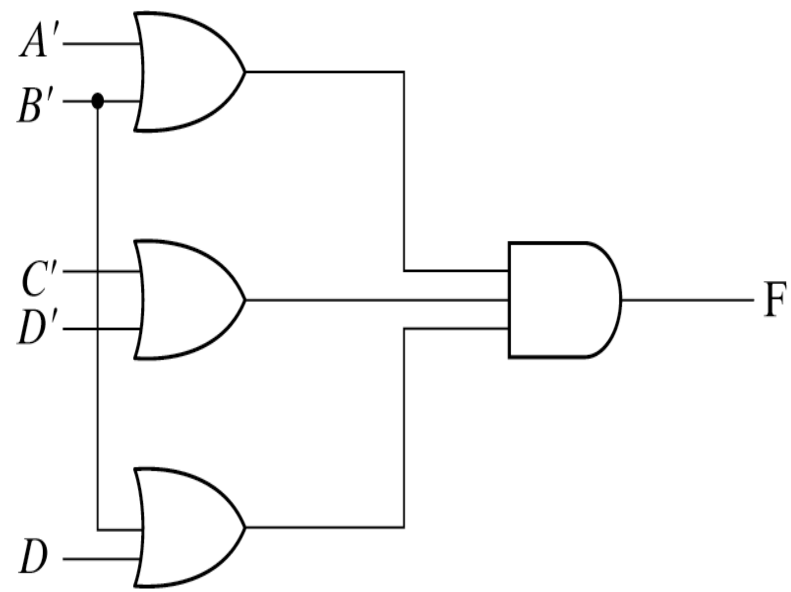
SOP and POS gate implementation

SUM OF PRODUCT (SOP)



(a) $F = B'D' + B'C' + A'C'D$

PRODUCT OF SUM (POS)



(b) $F = (A' + B')(C' + D')(B' + D)$

Fig. 3-15 Gate Implementation of the Function of Example 3-8