D.C.Generator

D.C.Machines

An emf is induced in a circuit placed in a magnetic field if either:

- the magnetic flux linking the circuit is time varying or
- there is a relative motion between the circuit and the magnetic field such that the conductors comprising the circuit cut a cross the magnetic flux lines.

- 1st form of the law is the basis of transformers.
- 2nd form is the basic principle of operation of electric generators

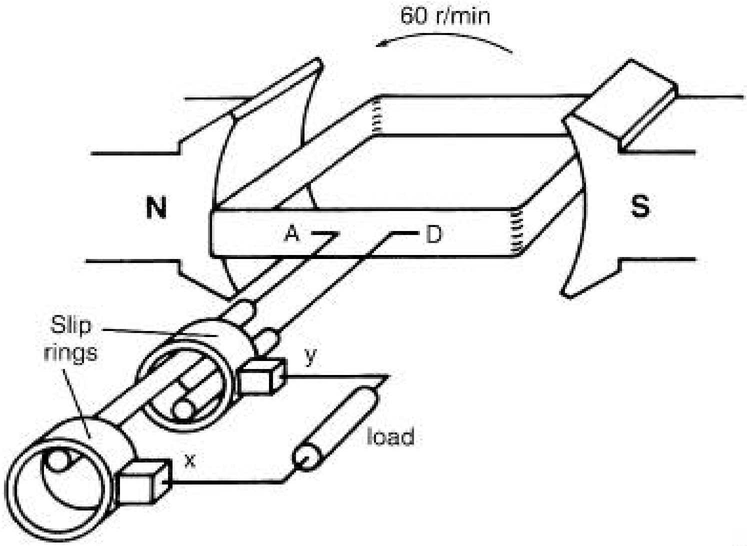


Figure 4.1
Schematic diagram of an elementary ac generator turning at 1 revolution per second.

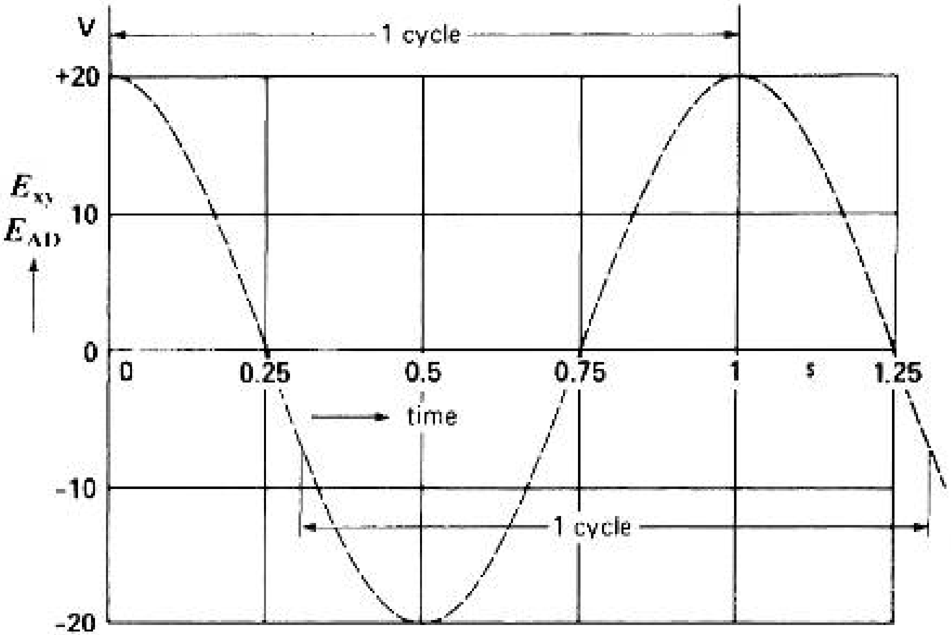
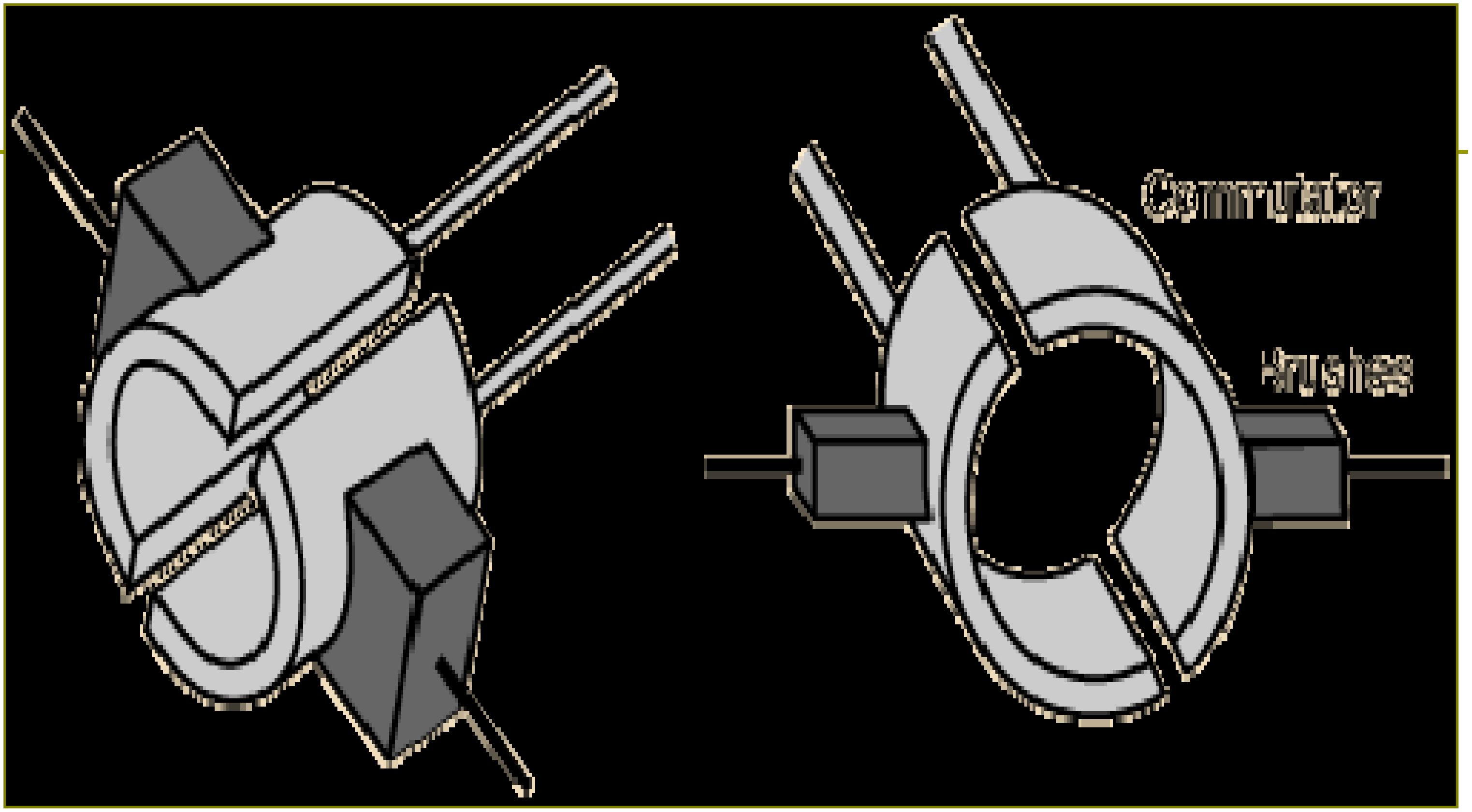
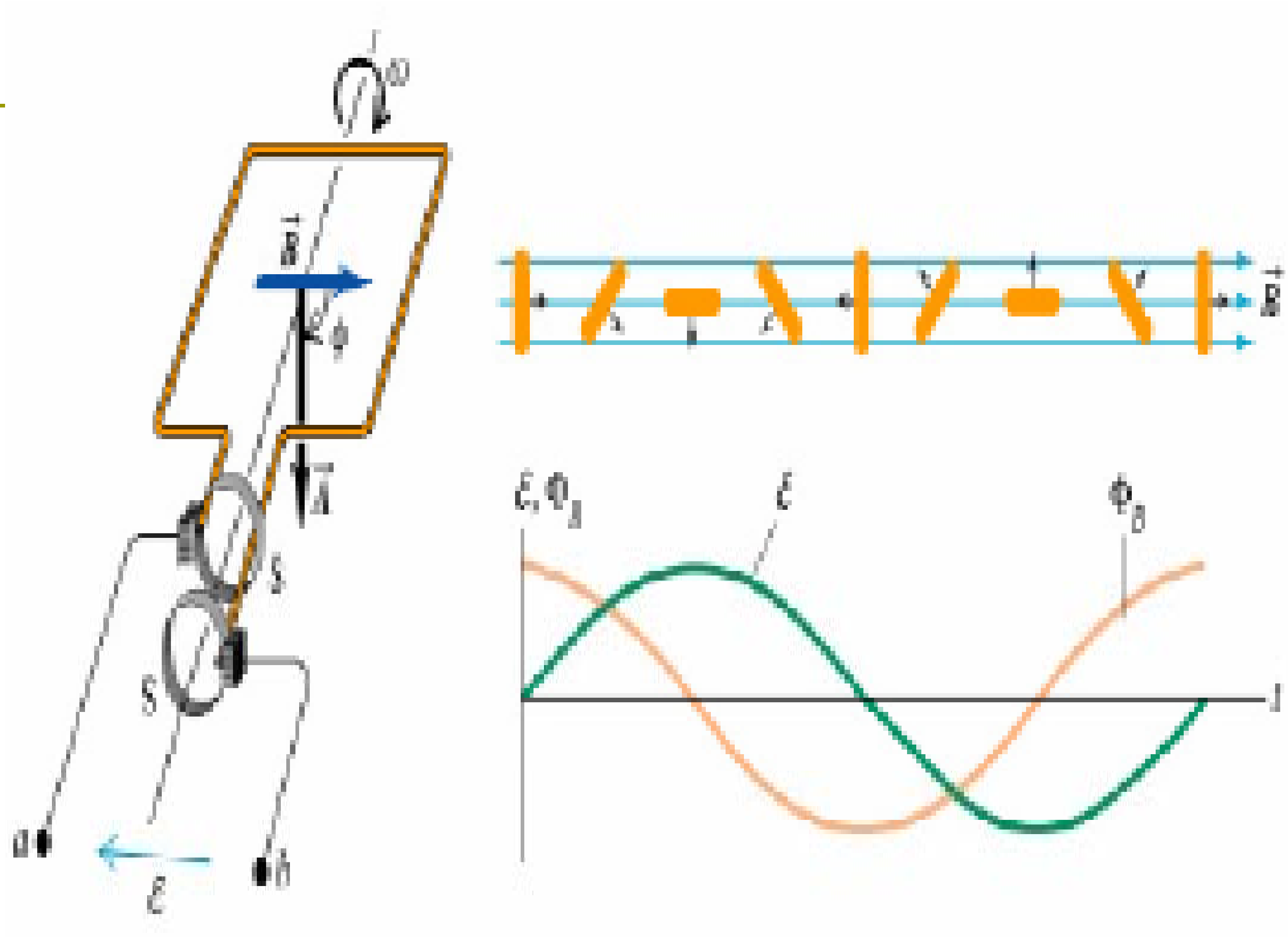


Figure 4.3
Voltage induced as a function of time.



The purpose of the brush is to ensure electrical connections between the rotating commutator and stationary external load circuit. It is made of carbon and rest on the commutator


ω
$\vec{B}$
φ
$\vec{A}$
S
S
a
b
ℰ
$\vec{B}$
ℰ, Φ_B
ℰ
Φ_B
t

DC Generator Operation

The N-S poles produce a dc magnetic field and the rotor coil turns in this field. A turbine or other machine drives the rotor. The conductors in the slots cut the magnetic flux lines, which induce voltage in the rotor coils. The coil has two sides:one is placed in slot a, the other in slot b.

ACTION OF A COMMUTATOR

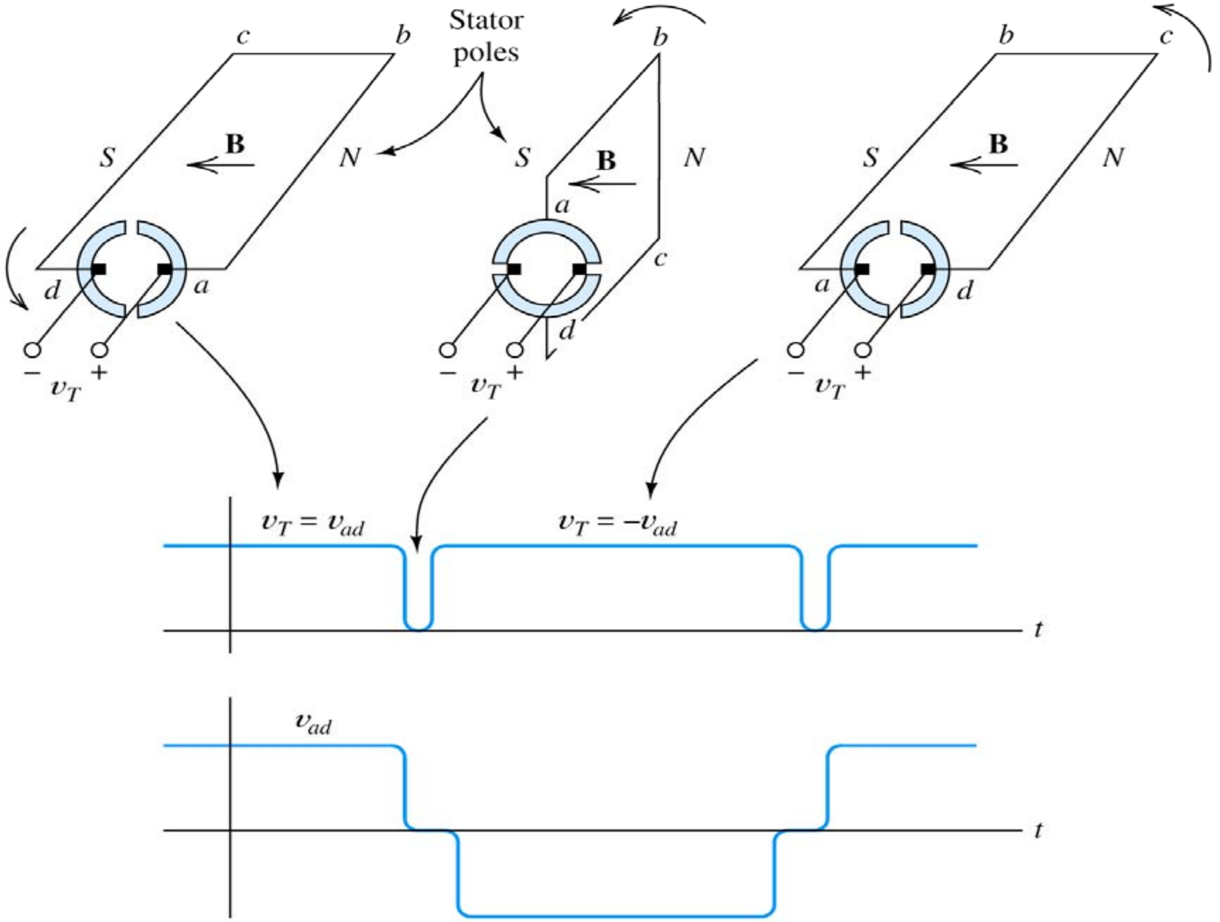


Figure 16.12 Commutation for a single armature winding.

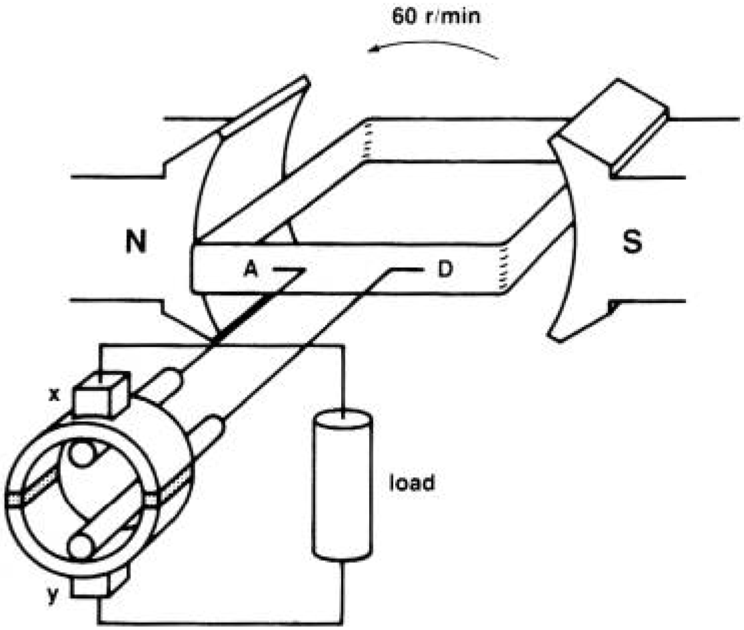


Figure 4.4
Elementary dc generator is simply an ac generator equipped with a mechanical rectifier called a *commutator.*

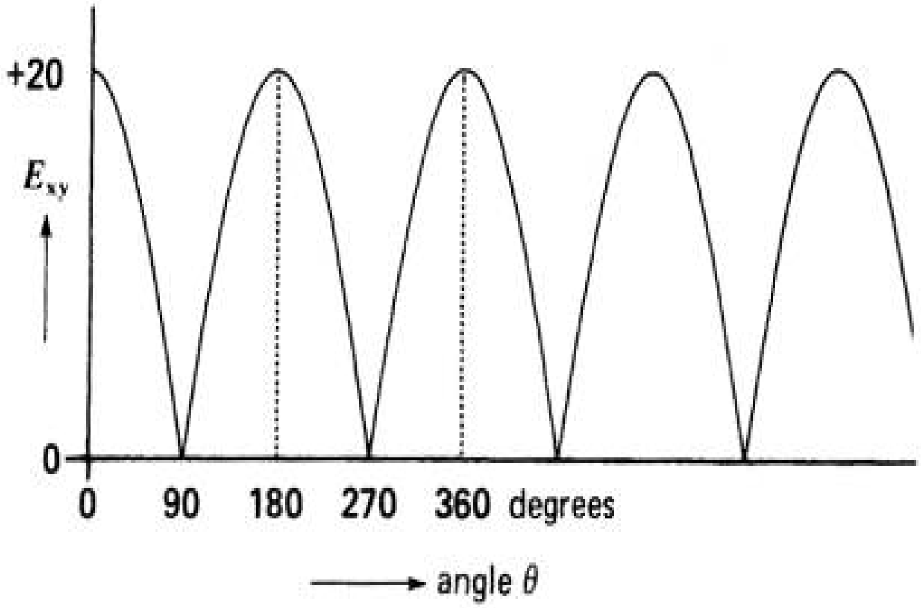


Figure 4.5
The elementary dc generator produces a pulsating dc voltage.

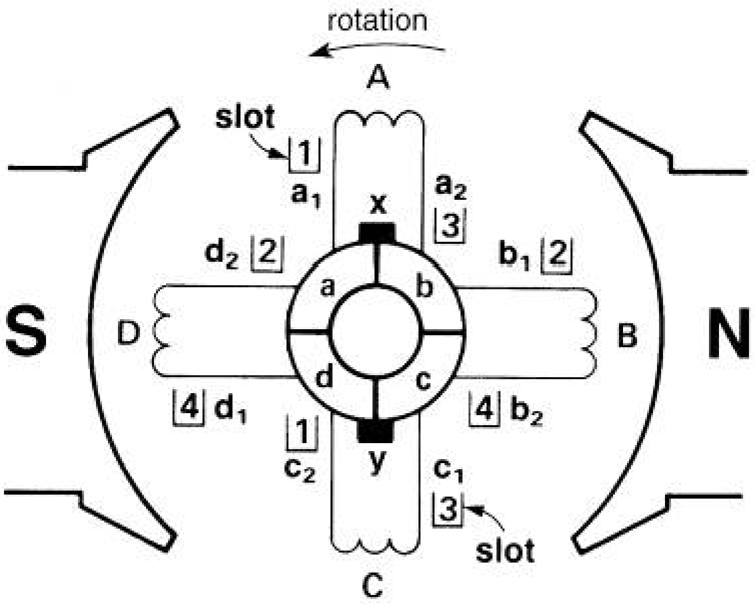


Figure 4.7
Schematic diagram of a dc generator having 4 coils and 4 commutator bars. See Fig. 4.9.

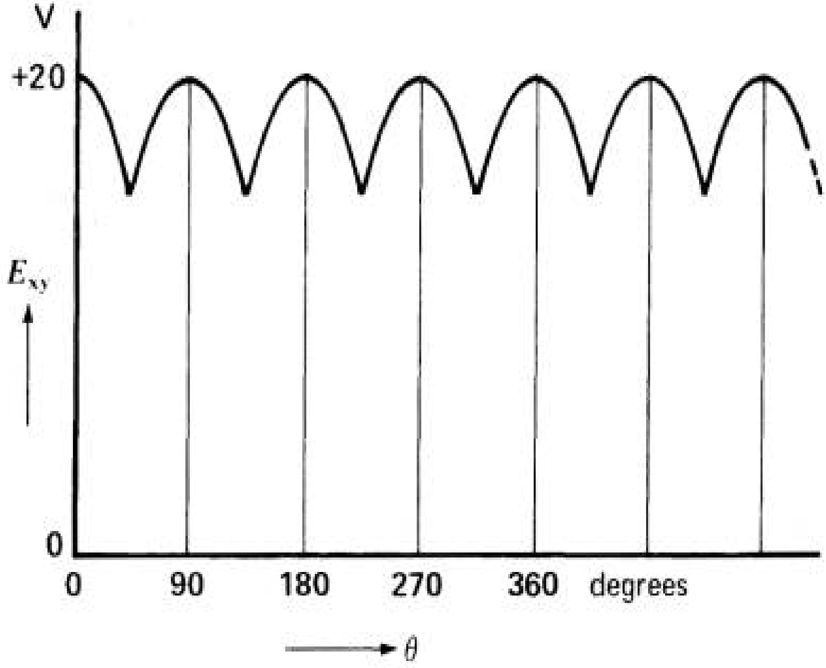


Figure 4.8
The voltage between the brushes is more uniform than in Fig. 4.5.

DC Generators: Operating Principle

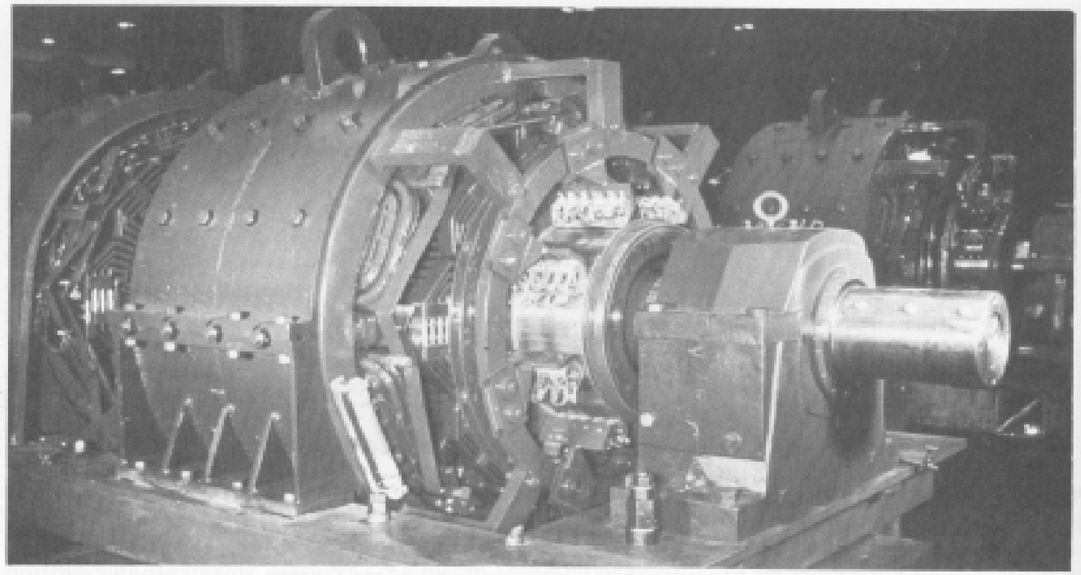
The difference between AC and DC generators:

- AC generators use *slip rings*
- DC generators use *commutators*

Otherwise, the machine constructions are essentially the same

DC Machines

A DC Machine



Armature along with the commutator

Significant Features of DC Machines

- Conventional DC generators are being replaced by the solid state rectifiers where ac supply is available.
- The same is not true for dc motors because of
    - Constant mechanical power output or constant torque
    - Rapid acceleration or deceleration
    - Responsiveness to feedback signals
- 1W to 10,000 hp

Introduction

Electromagnetic Energy Conversion:

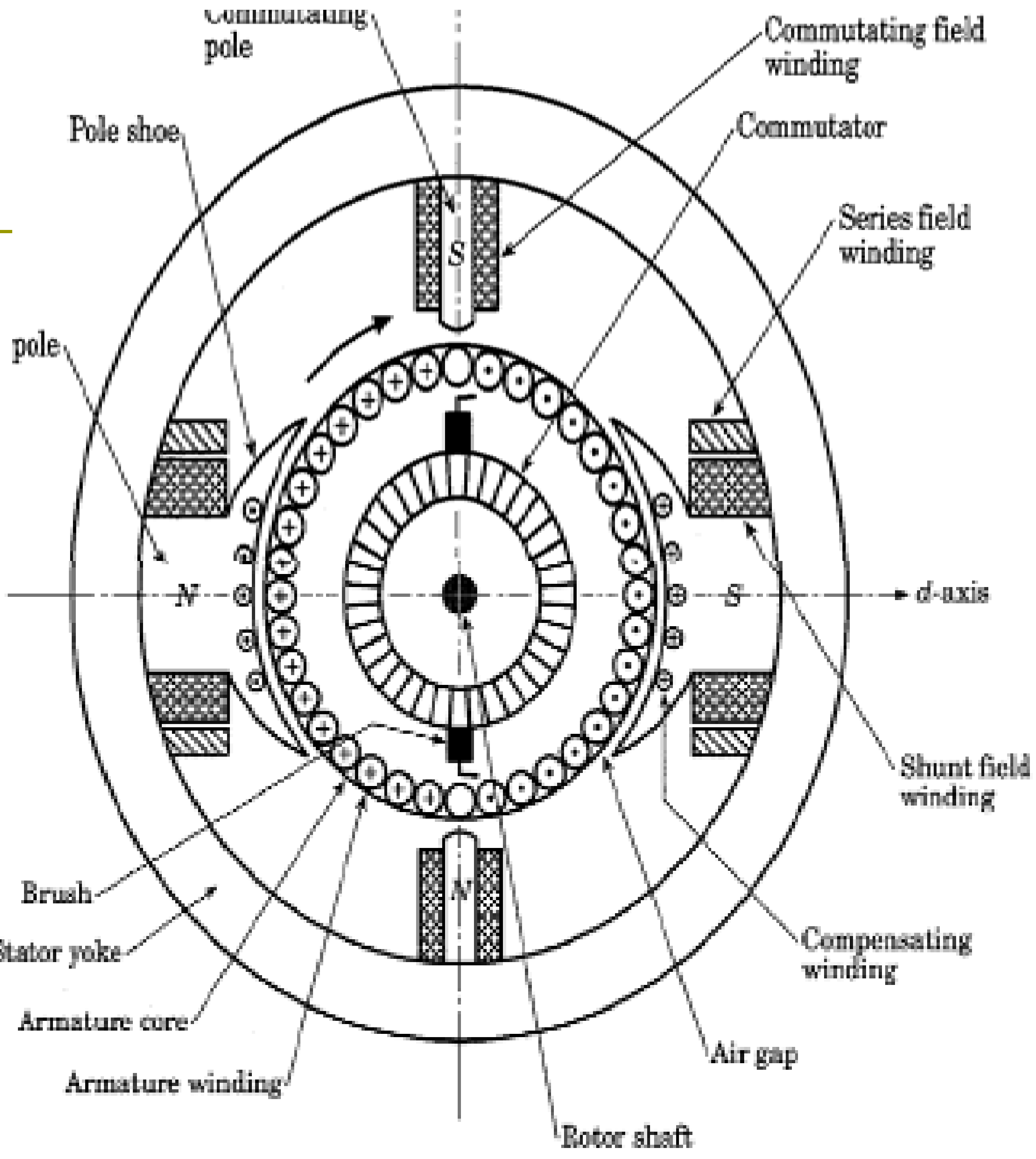
1. When armature conductors move in a magnetic field produced by the current in stator field winding, voltage is induced in the armature conductors.
2. When current carrying armature conductors are placed in a magnetic field produced by the current in stator field winding, the armature conductors experience a mechanical force.

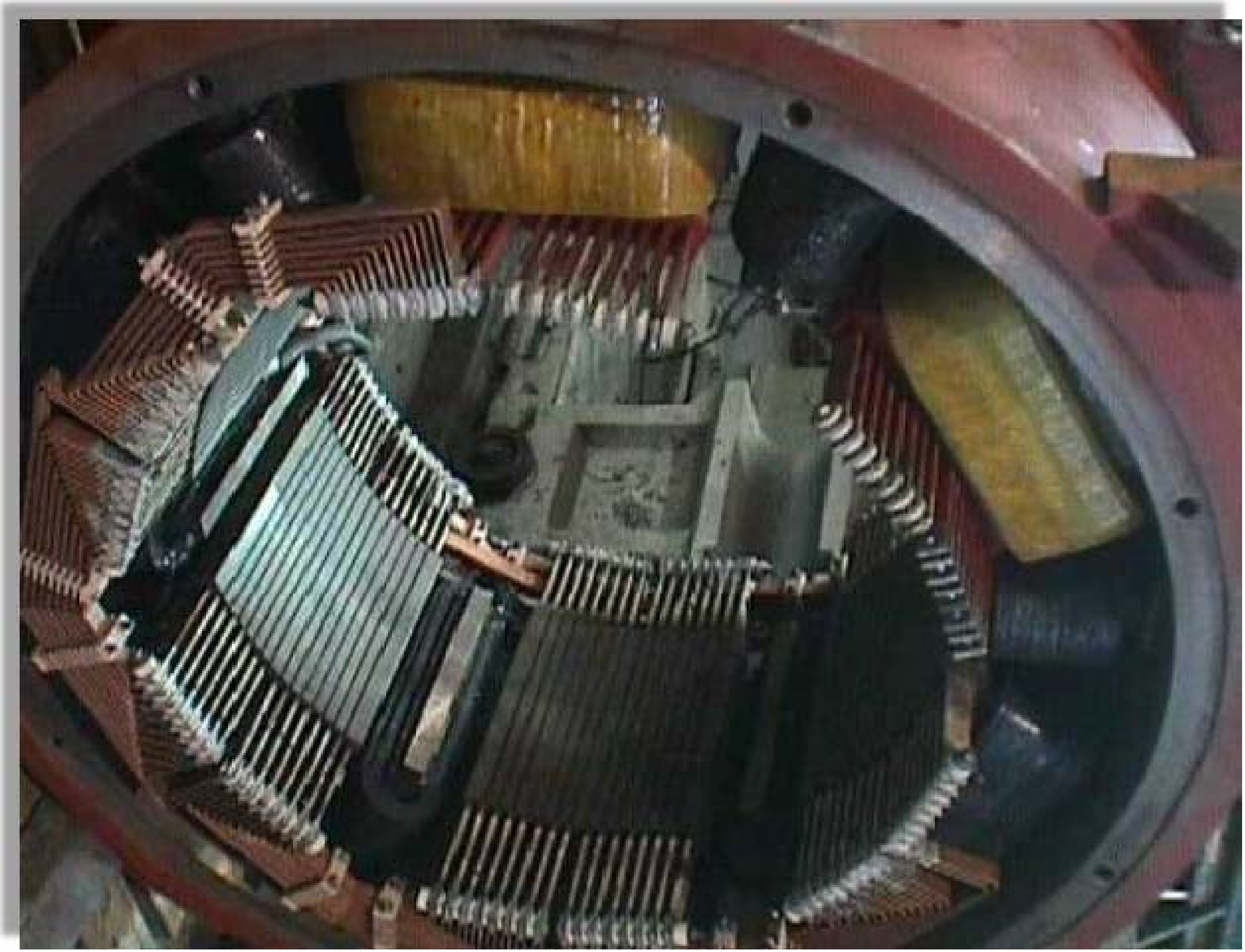
These two effects occur simultaneously in a DC machine whenever energy conversion takes place from electrical to mechanical or vice versa.

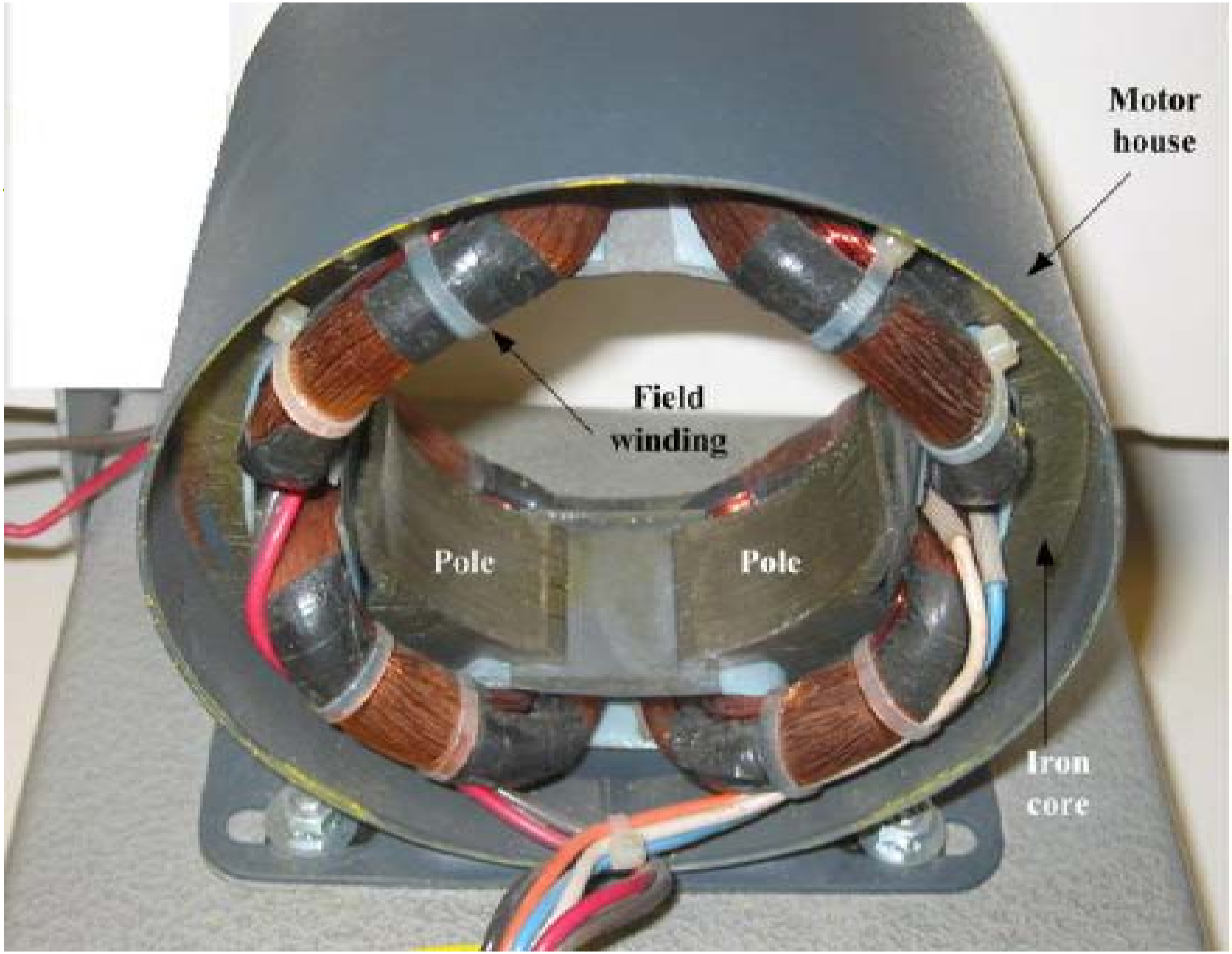
Construction

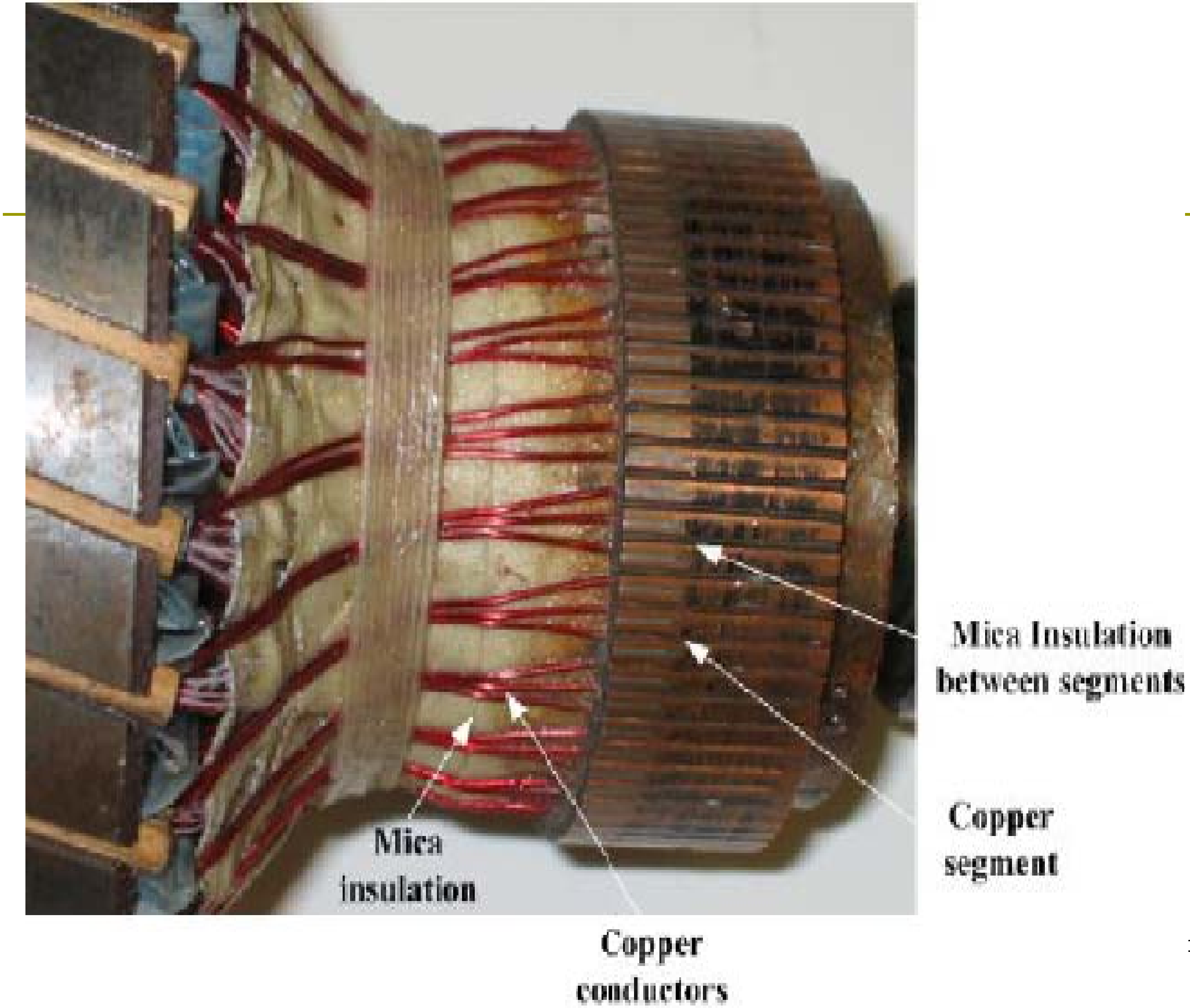
- Field Magnets
- Yoke
- Pole, pole shoes, magnetising coils
- Armature
- Brush
- Commutator
- Compensating winding
- Interpoles (Compoles)

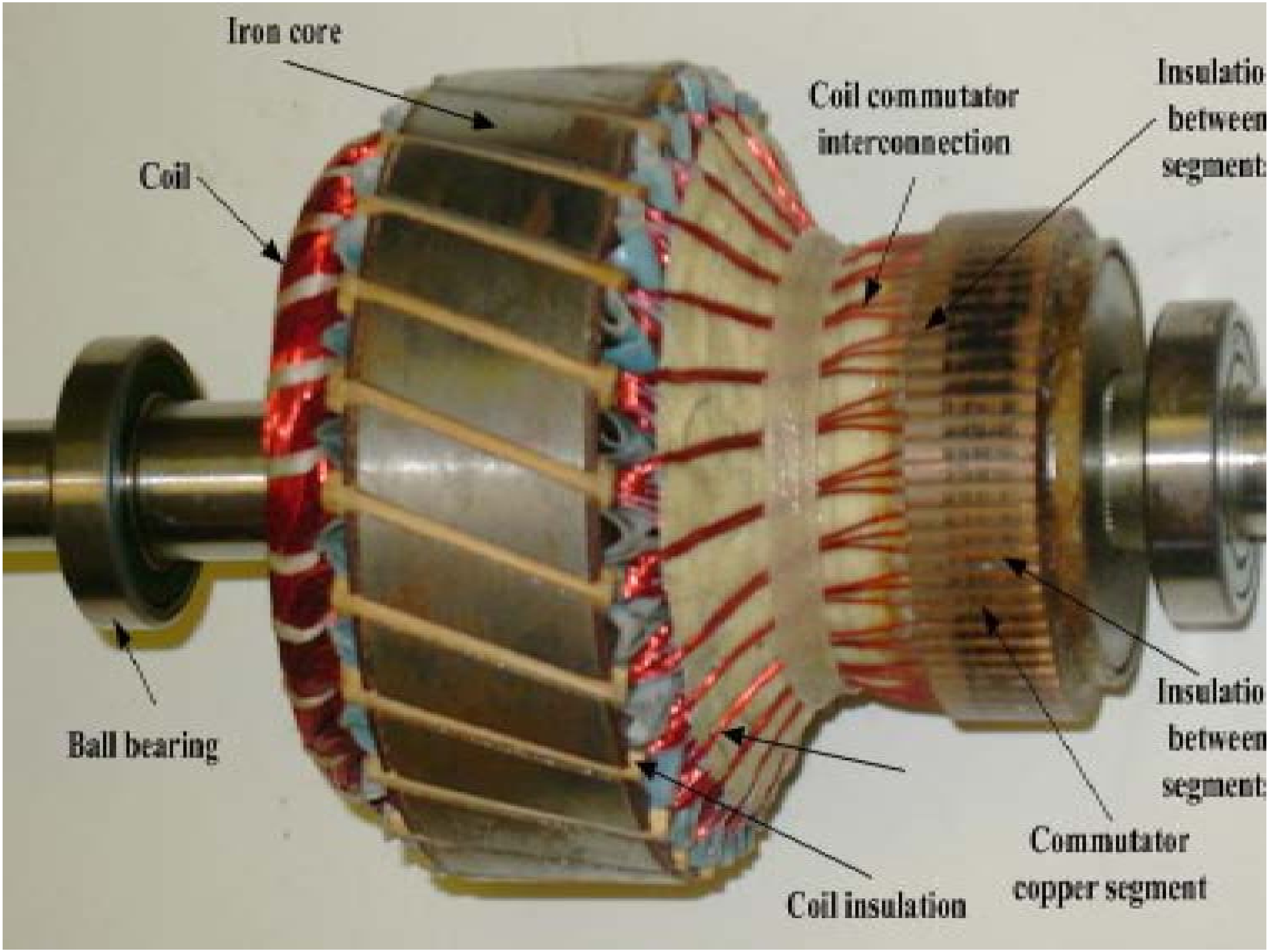
CONSTRUCTION

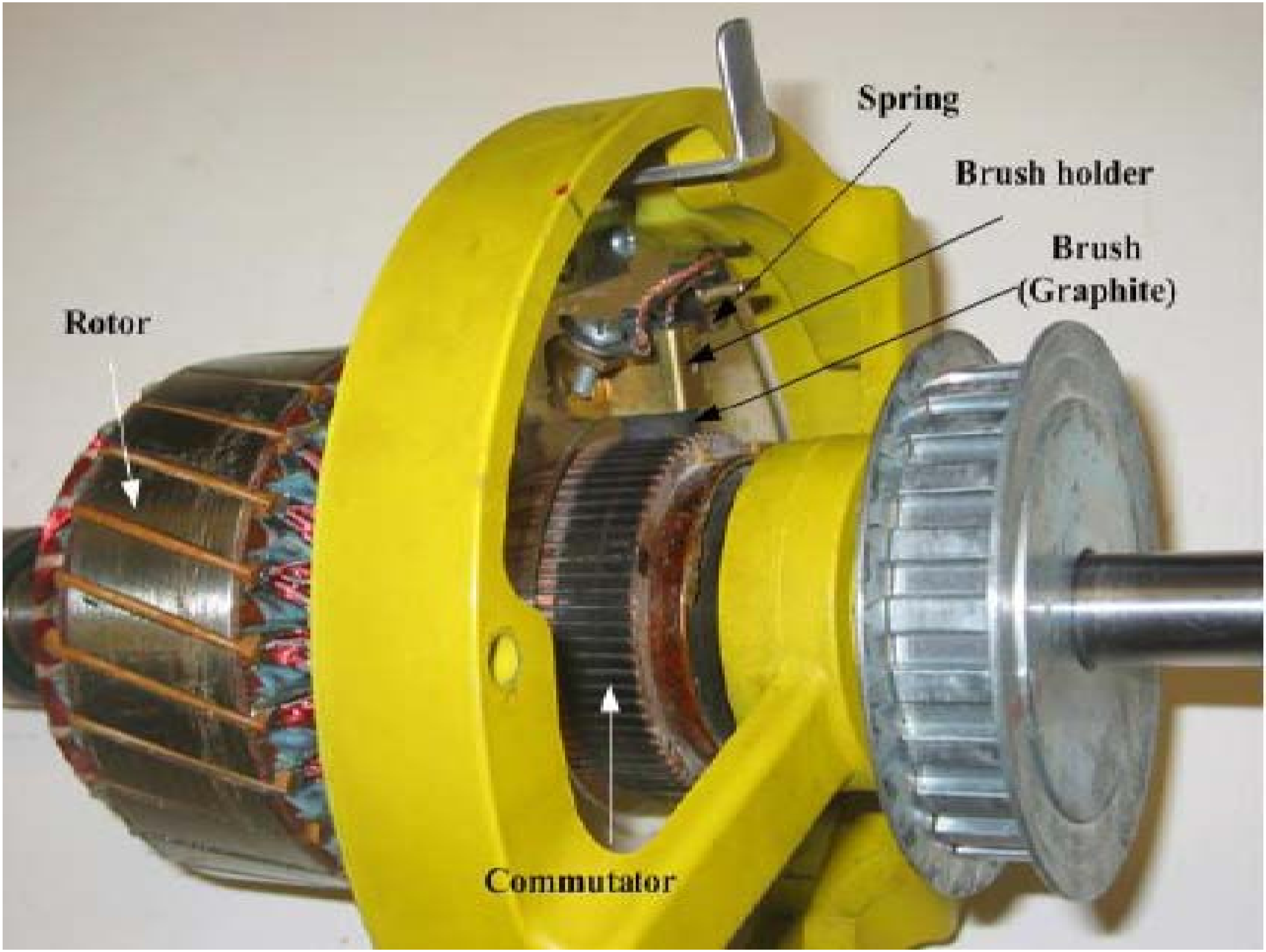

Commutating
pole
Pole shoe
pole
Commutating field
winding
Commutator
Series field
winding
S
N
N
S
d-axis
Shunt field
winding
Brush
Stator yoke
Armature core
Armature winding
Compensating
winding
Air gap
Rotor shaft




Motor house
Field winding
Pole
Pole
Iron core


Mica Insulation between segments
Copper segment
Mica insulation
Copper conductors


Iron core
Coil commutator interconnection
Insulatio between segment
Coil
Ball bearing
Insulatio between segment
Commutator copper segment
Coil insulation


Spring
Brush holder
Brush
(Graphite)
Rotor
Commutator

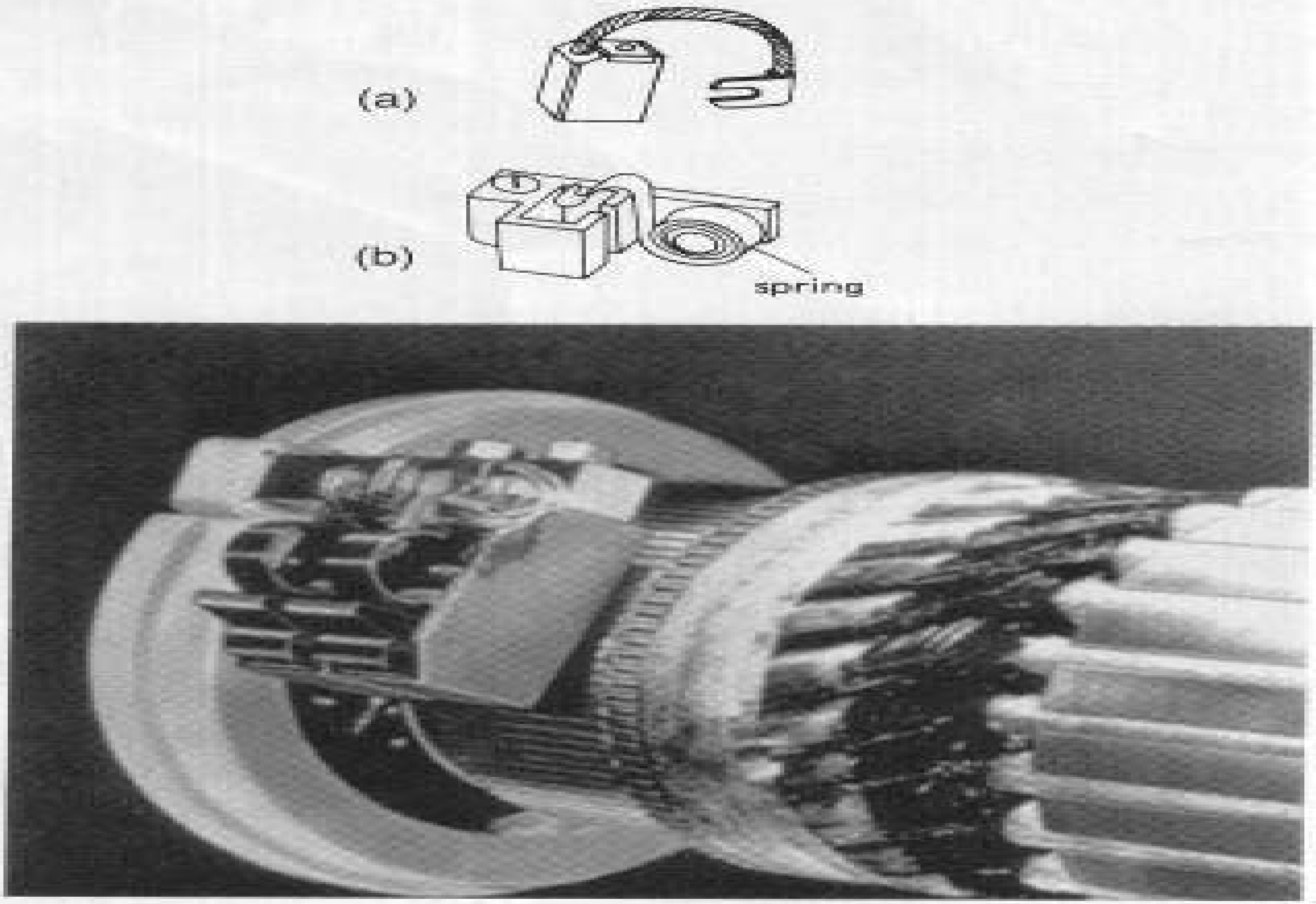


Figure 4.35

a. Carbon brush and ultraflexible copper lead.
b. Brush holder and spring to exert pressure.
c. Brush set composed of two brushes, mounted on rocker arm.

(Courtesy of General Electric Company, USA)

ARMATURE OF A DC MOTOR

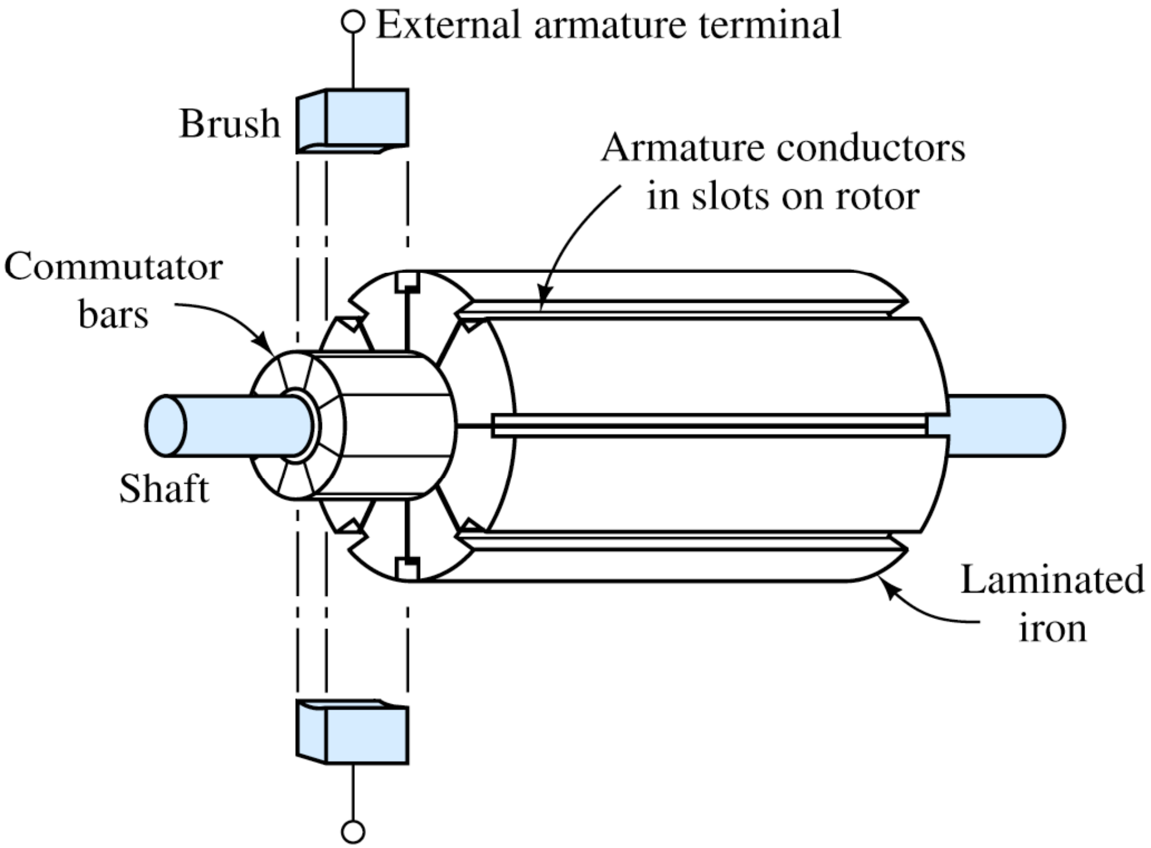


Figure 16.9 Rotor assembly of a dc machine.

ROTOR CONSTRUCTION:

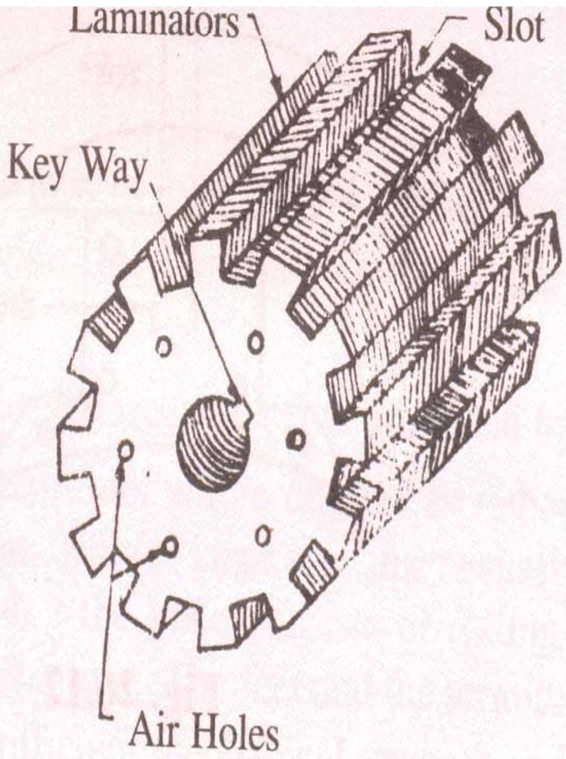


Fig. 26.15

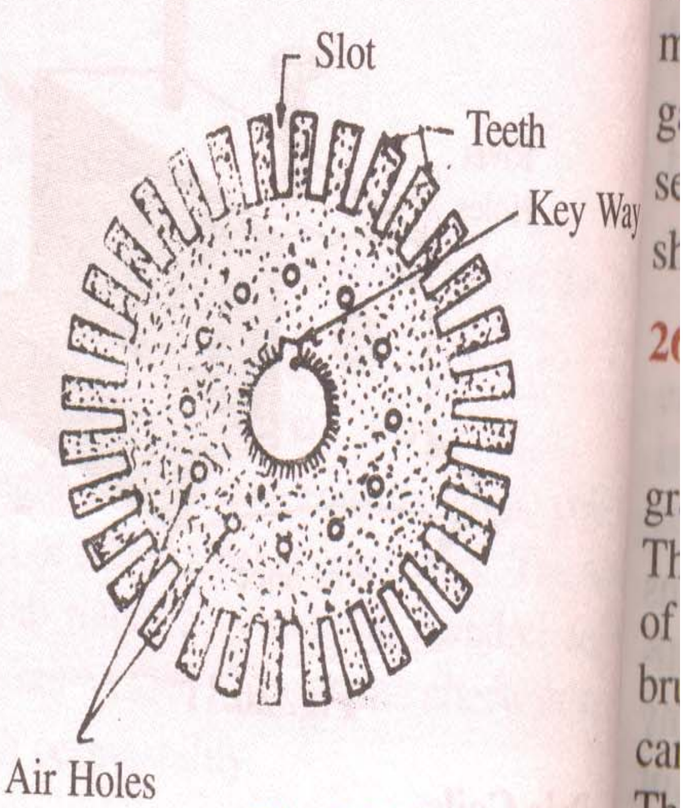


Fig. 26.16

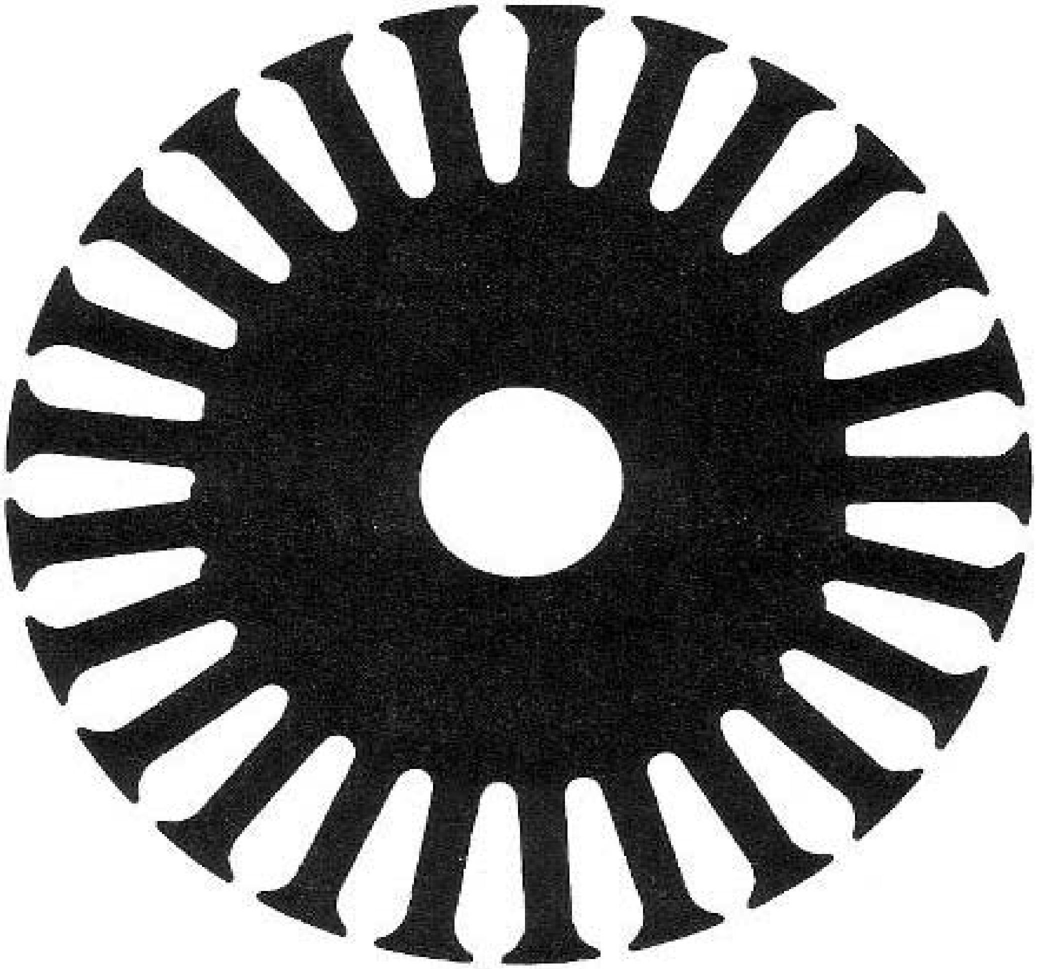


Figure 4.31
Armature lamination with tapered slots.

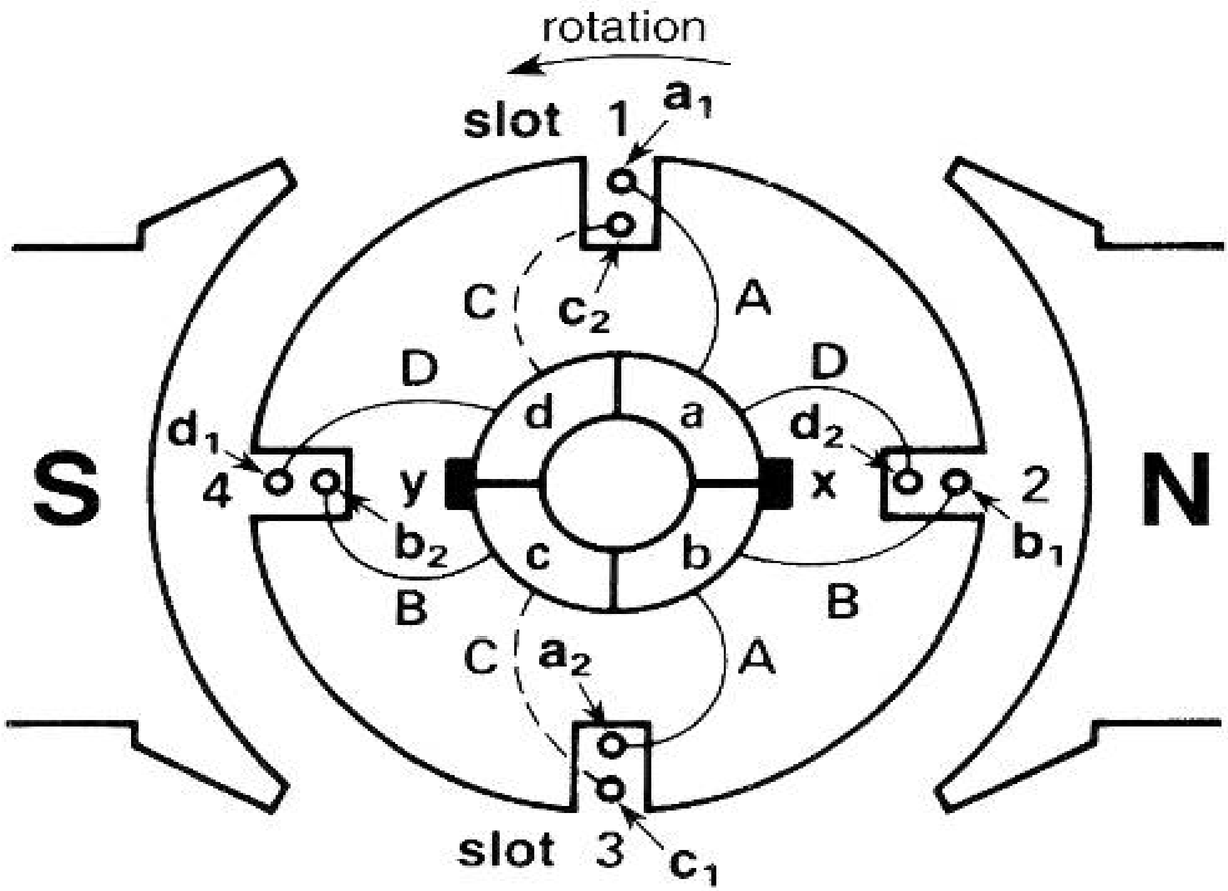


Figure 4.9
The actual physical construction of the generator shown in Fig. 4.7. The armature has 4 slots, 4 coils, and 4 commutator bars.

Effects of armature reaction and remedy

- Armature reaction distorts the flux density distribution, produces demagnetizing effect and shifts the zero flux density region from the q-axis.
- Poor commutation and sparking.

Rotor mmf neutralized by

- compensating winding fitted on the main pole faces and connected in series with armature winding.
- Armature current reversal is delayed due to coil inductance and reactance voltage ie. voltage induced in the moving coil by the quadratic flux.
- inter-pole is needed to compensate armature reaction

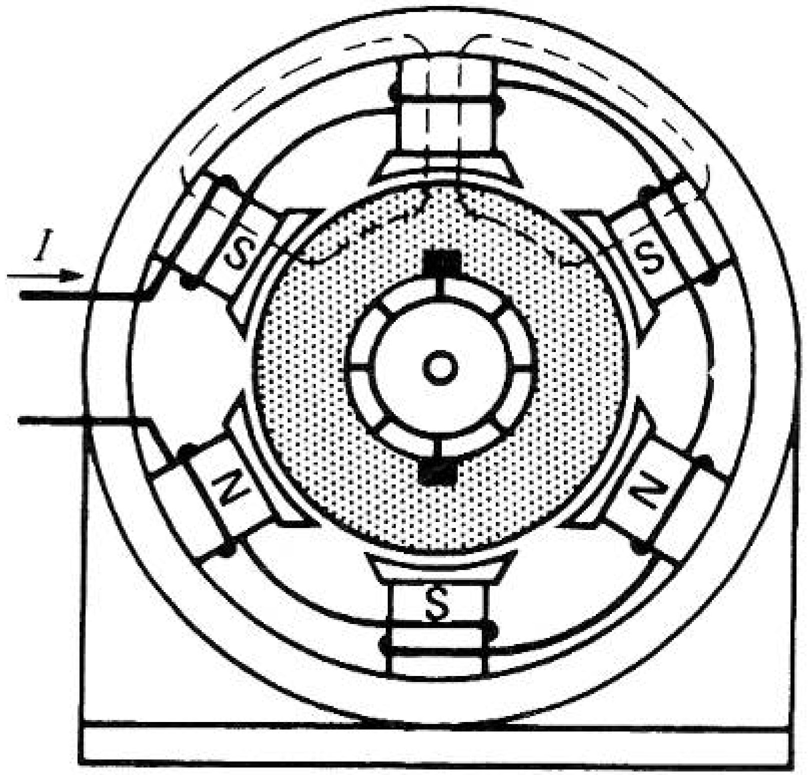


Figure 4.29

Adjacent poles of multipole generators have opposite magnetic polarities.

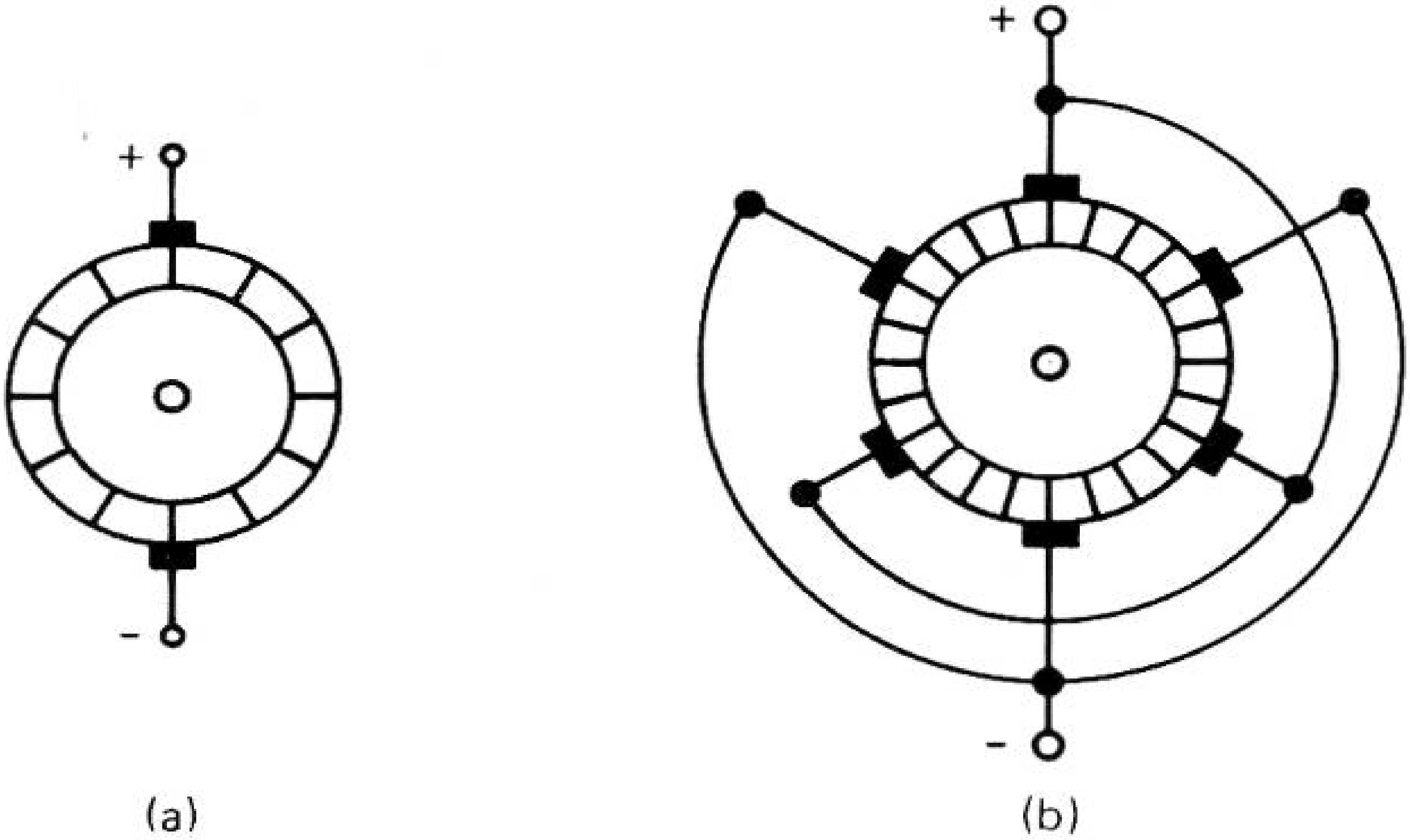


Figure 4.34

a. Brushes of a 2-pole generator.

b. Brushes and connections of a 6-pole generator.

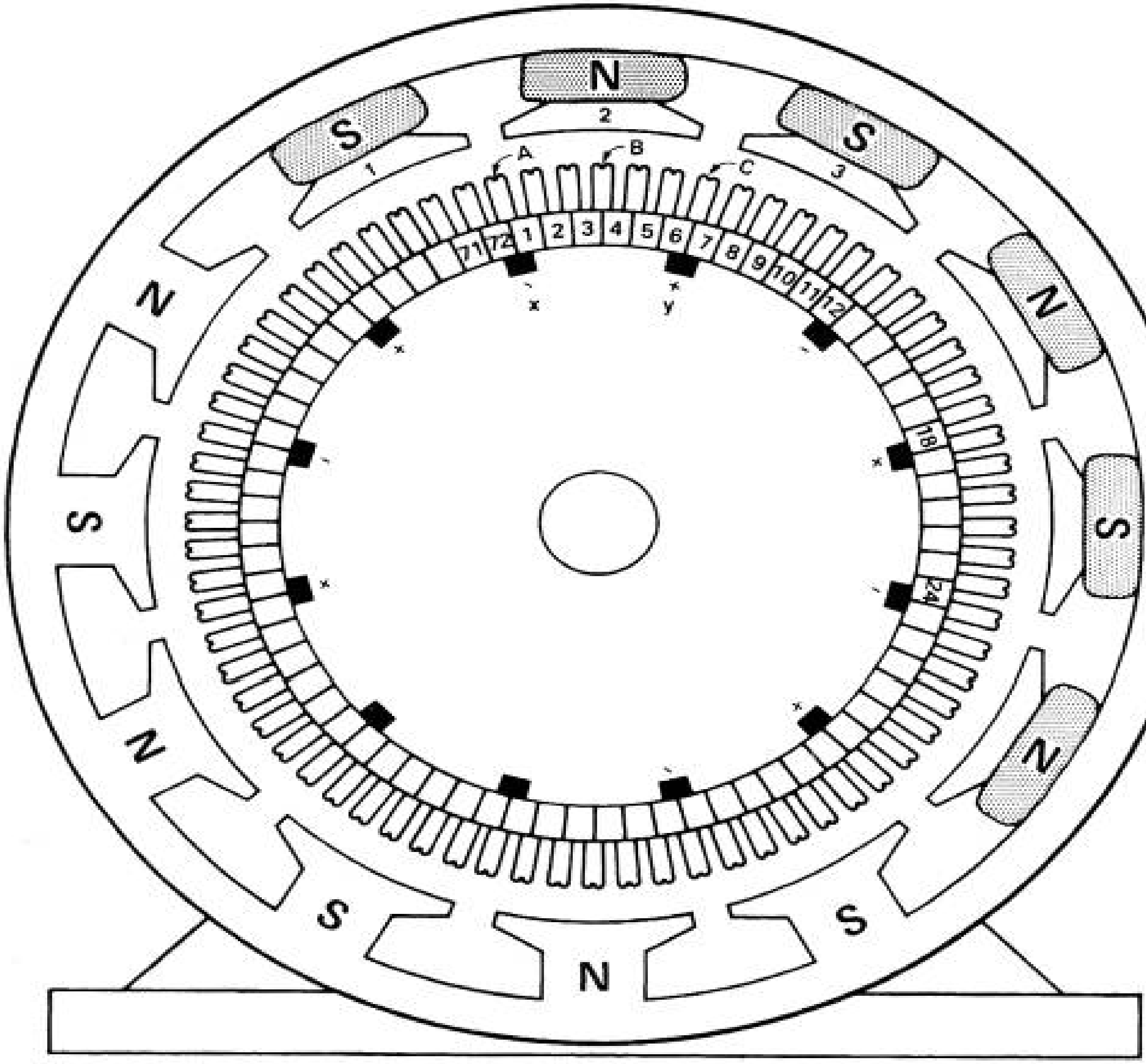


Figure 4.38a
Schematic diagram of a 12-pole, 72-coil dc generator.

ROTOR WINDING:

- Rotor windings are of two types:
  - Lap winding
  - Wave winding

- No. of parallel paths:

  In lap winding p= no. of poles

  In wave winding p= 2

Principle of Operation: Armature Voltage

- $$\text{Emf}_{conductor} = \frac{\text{Flux/rev.}}{\text{Time/rev.}} = \frac{p.\text{ø}}{60/N_m} = \frac{p.\ \text{ø}.\ N_m}{60}$$

- *Emf*

Emf_{total}=Emf conductor X Number of conductor path

$$\text{Emf}_{total} = \frac{p.\ \text{ø}.\ N_m}{60}(Z/A) = \frac{ZNP\text{ø}}{60\ A} \text{.........}$$

where

- *p* = number of poles
- *Z* = total number of armature conductors
- *A* = number of parallel paths, *2 for wave and p for lap*.
- Φ = flux per pole (Weber)
- *Nm* = speed of the armature in the revolutions per minute (rpm)
- time of 1 revolution = 60/Nm (sec)

Types of DC generators

There are four different methods for supplying the dc current to the motor or generator poles:

- **– Separate excitation**
- **– Shunt connection**
- **– Series connection**
- **– Compound**

Types of DC Machines

Both the armature and field circuits carry direct current in the case of a DC machine.

Types:

Self-excited DC machine: when a machine supplies its own excitation of the field windings. In this machine, residual magnetism must be present in the ferromagnetic circuit of the machine in order to start the self-excitation process.

Separately-excited DC machine: The field windings may be separately excited from an eternal DC source.

Shunt Machine: armature and field circuits are connected in parallel. Shunt generator can be separately-excited or self-excited.

Series Machine: armature and field circuits are connected in series.

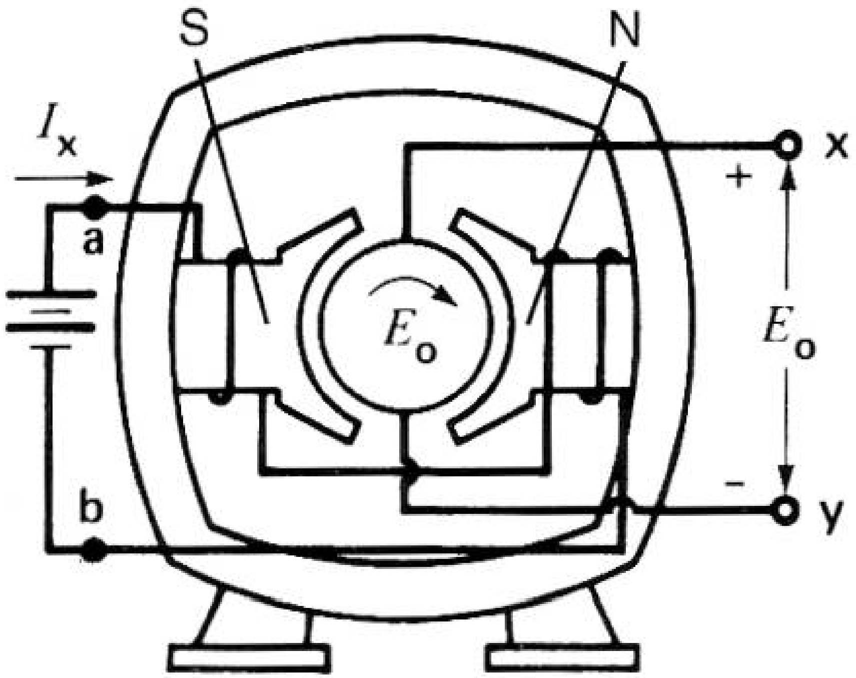
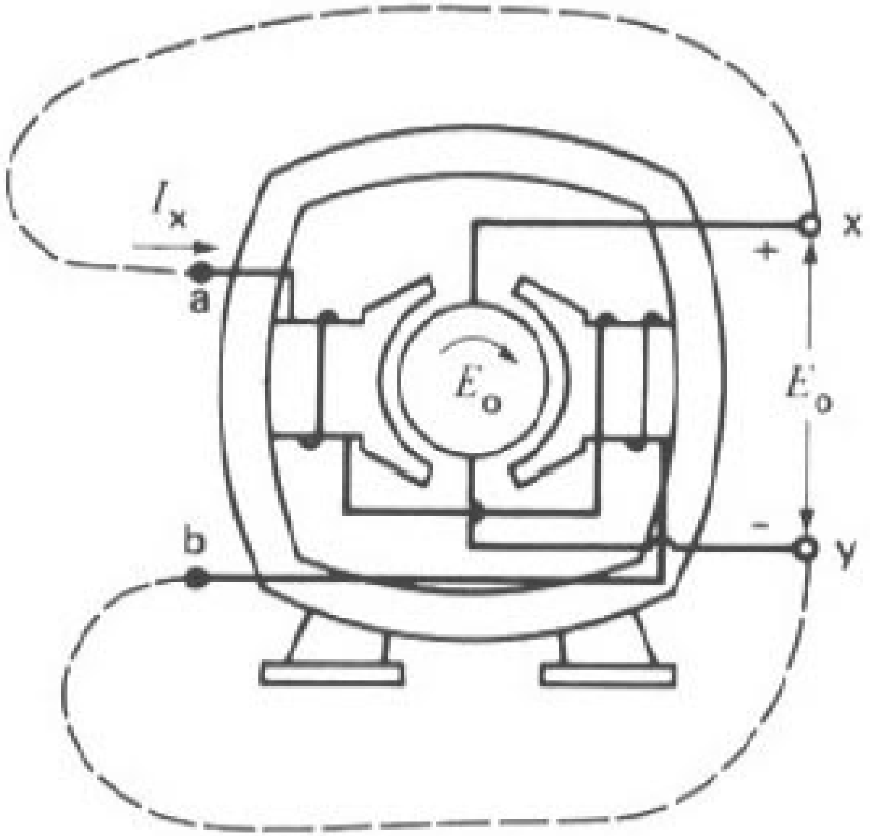
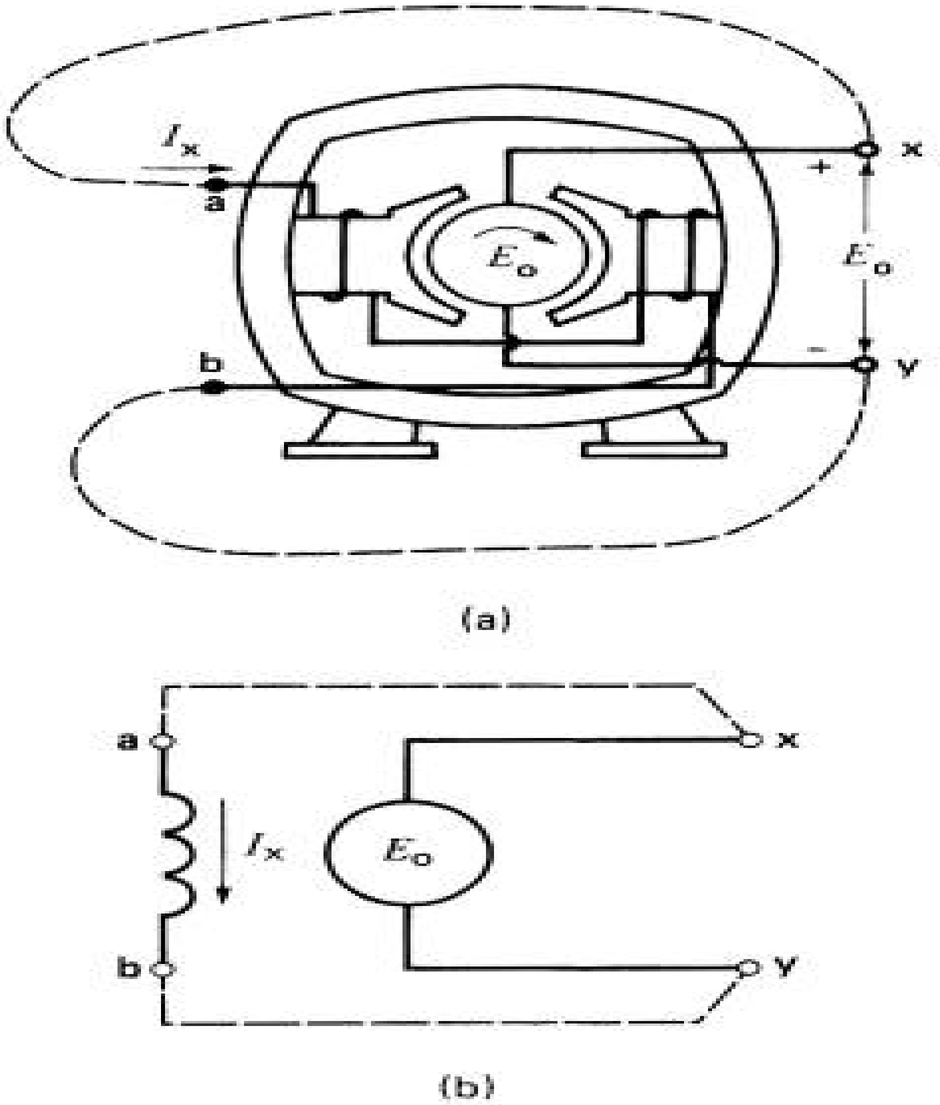


Figure 4.17
Separately excited 2-pole generator. The N, S field poles are created by the current flowing in the field windings.

Shunt DC Generator





(a)

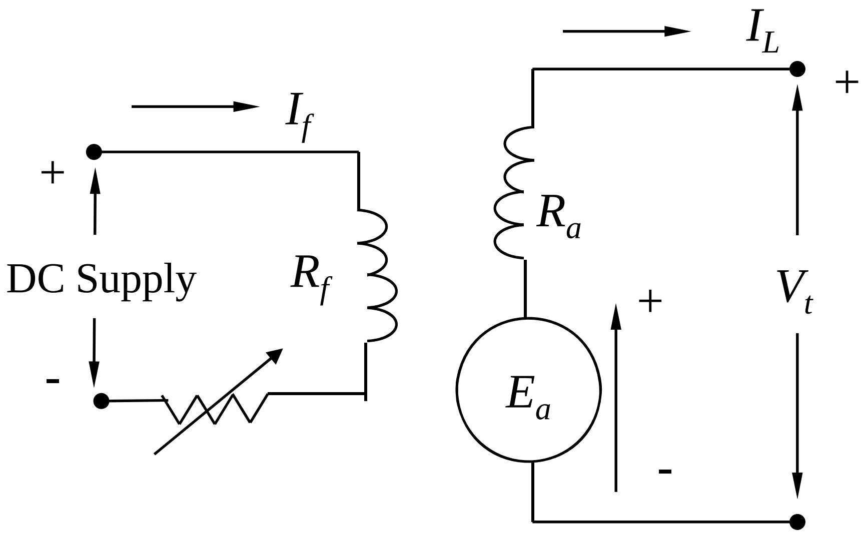
(b)

Figure 4.19

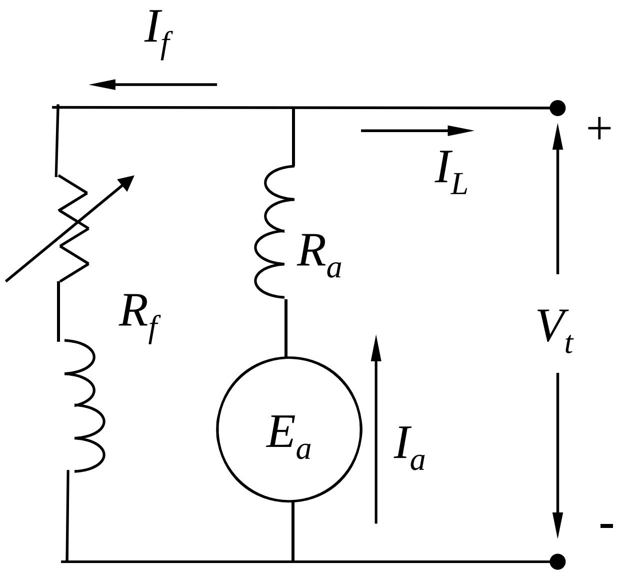
a. Self-excited shunt generator.

b. Schematic diagram of a shunt generator. A shunt field is one designed to be connected in shunt (alt nate term for parallel) with the armature winding.

Separately-Excited and Self-Excited DC Generators

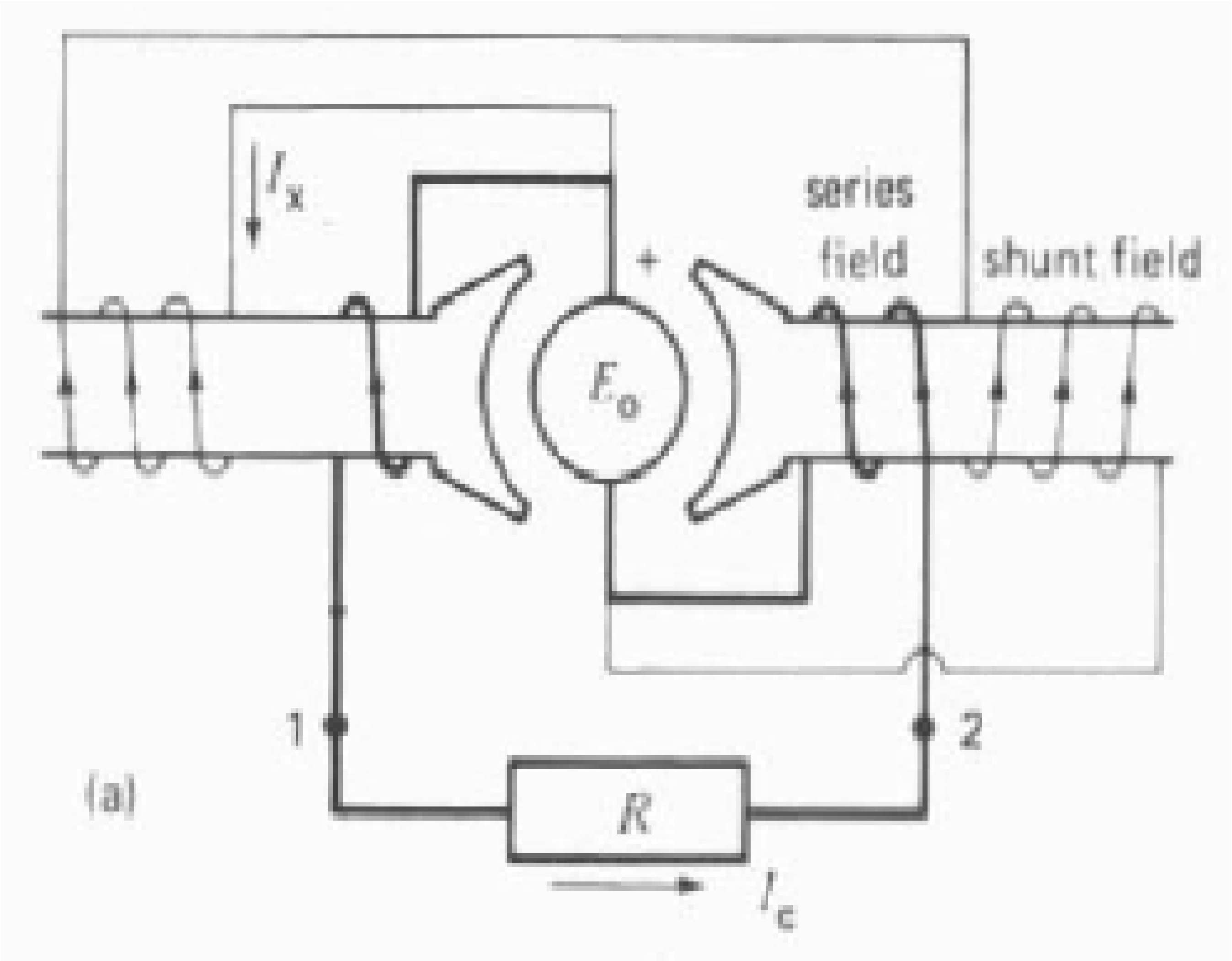


Separately-Excited



Self-Excited

Compound DC Generator



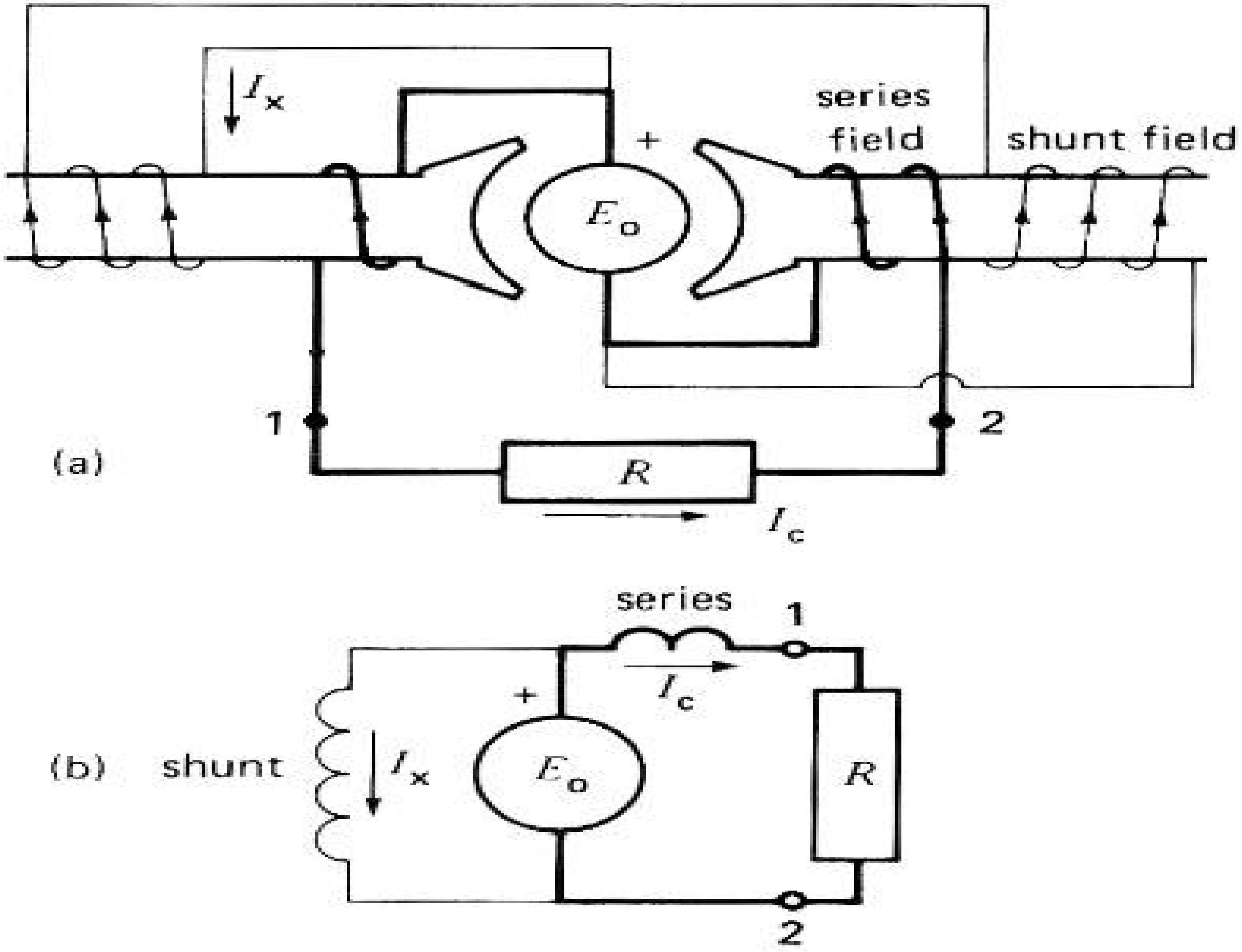
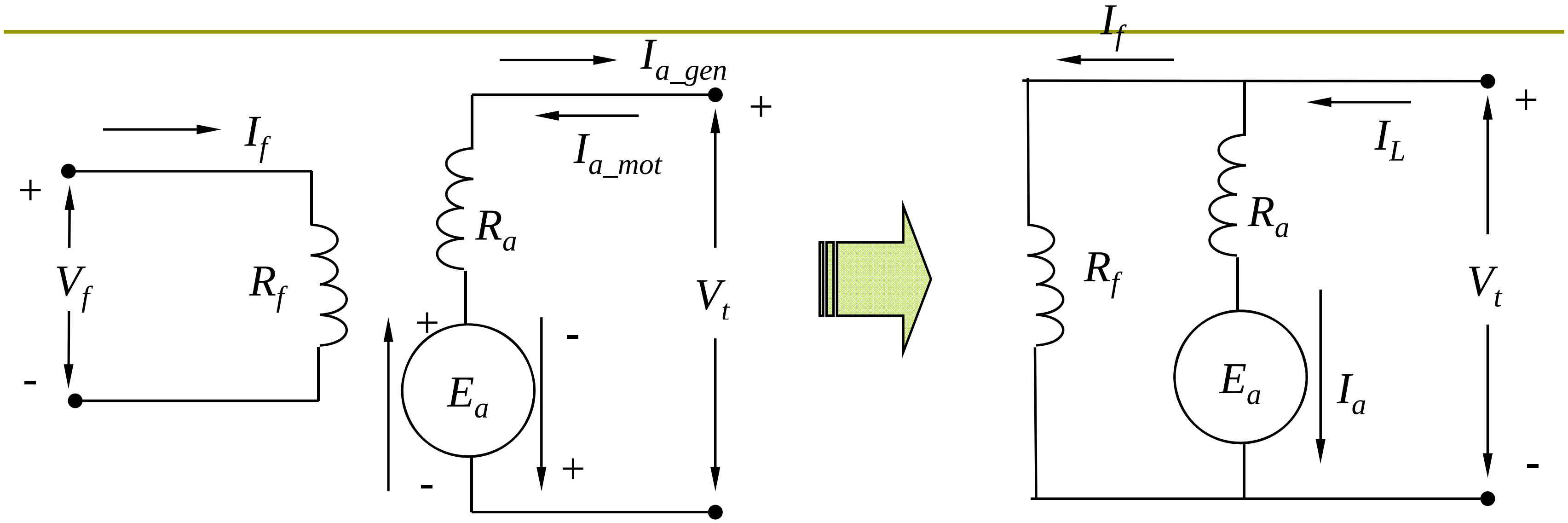


Figure 4.25
a. Compound generator under load.
b. Schematic diagram.

Equivalent Circuit of a DC Machine



$$\boxed{\begin{aligned} V_f &= I_f R_f \\ V_t &= E_a \pm I_a R_a \end{aligned}}$$

Generated *emf* and Electromagnetic Torque

$$\boxed{\begin{array}{l} V_f = I_f R_f \\ V_t = E_a \pm I_a R_a \end{array}}$$

Motor: $V_t > E_a$
Generator: $V_t < E_a$

Voltage generated in the armature circuit due the flux of the stator field current

$$\boxed{E_a = K_a \phi_d \omega_m}$$

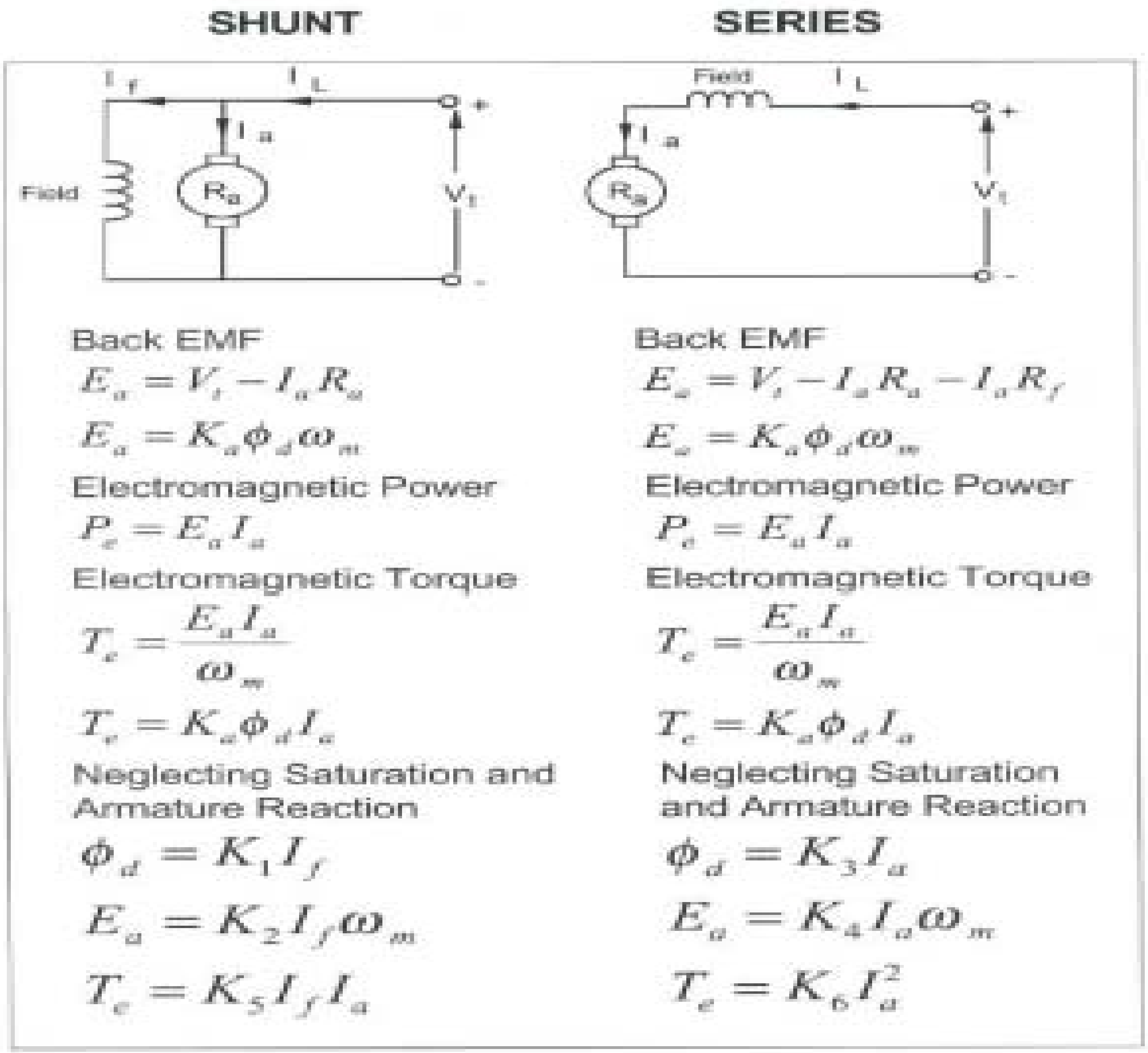
K_a: design constant

Electromagnetic torque

$$\boxed{T_e = K_a \phi_d I_a}$$

$$\boxed{P_{em} = E_a I_a = T_e \omega_m}$$

Comparison between the Shunt and Series Connected DC Machines



SHUNT	SERIES
Back EMF	Back EMF
$E_a = V_t - I_a R_a$	$E_a = V_t - I_a R_a - I_a R_f$
$E_a = K_a \phi_d \omega_m$	$E_a = K_a \phi_d \omega_m$
Electromagnetic Power	Electromagnetic Power
$P_e = E_a I_a$	$P_e = E_a I_a$
Electromagnetic Torque	Electromagnetic Torque
$T_e = \frac{E_a I_a}{\omega_m}$	$T_e = \frac{E_a I_a}{\omega_m}$
$T_e = K_a \phi_d I_a$	$T_e = K_a \phi_d I_a$
Neglecting Saturation and Armature Reaction	Neglecting Saturation and Armature Reaction
$\phi_d = K_1 I_f$	$\phi_d = K_3 I_a$
$E_a = K_2 I_f \omega_m$	$E_a = K_4 I_a \omega_m$
$T_e = K_5 I_f I_a$	$T_e = K_6 I_a^2$

Armature Reaction

1. It is the term used to describe the effects of the armature mmf on the operation of a dc machine as a "generator" no matter whether it is a generator or both the flux distribution and the flux magnitude in motor.
2. It effects the machine.
3. The distortion of the flux in a machine is called armature reaction

- Two effects of armature reaction:
  1. Neutral Plane Shift
  2. Flux Weakening

Armature Reaction

If a load is connected to the terminals of the dc machine, a current will flow in its armature windings. This current flow will produce a magnetic field of its own, which will distort the original magnetic field from the machine's field poles. This distortion of the magnetic flux in a machine as the load is increased is called the armature reaction.

ACTION OF A COMMUTATOR

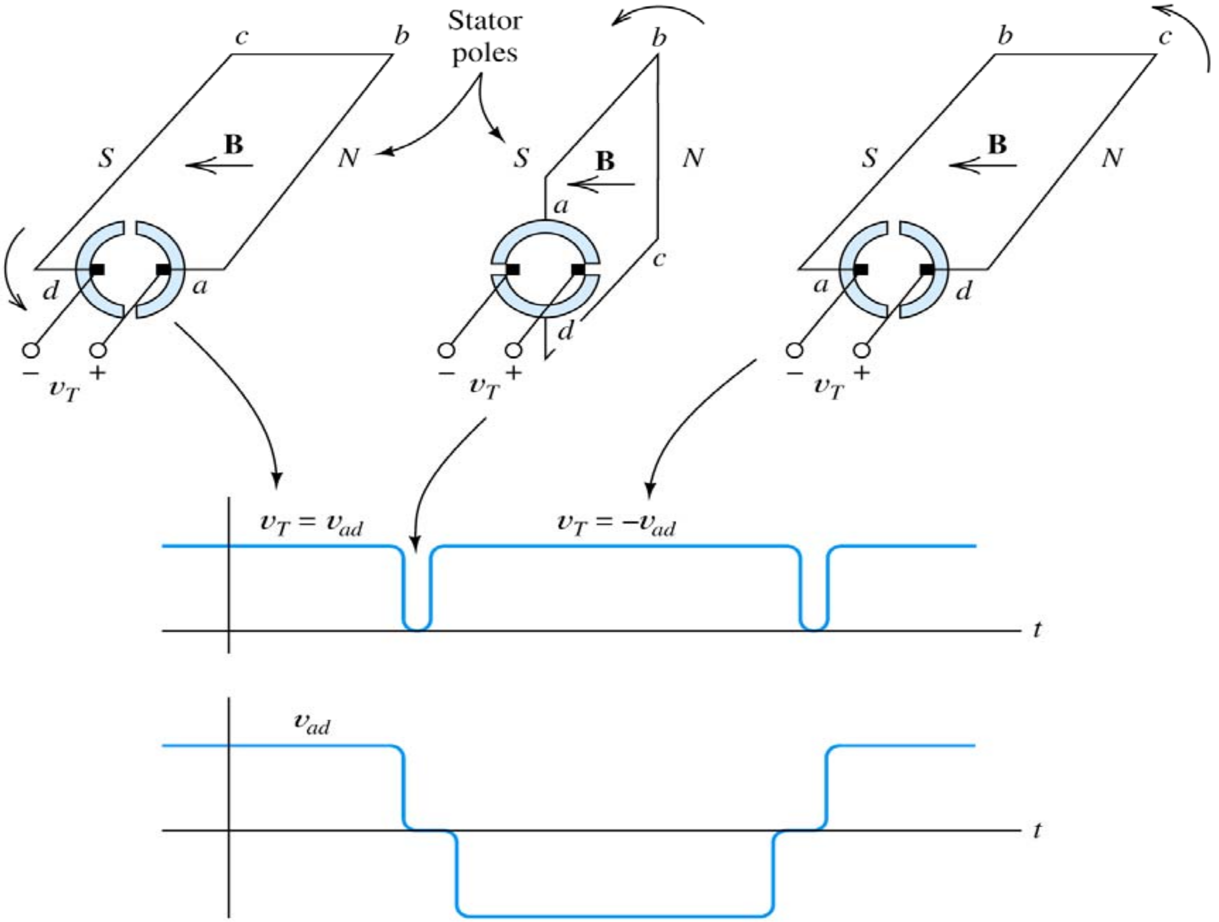
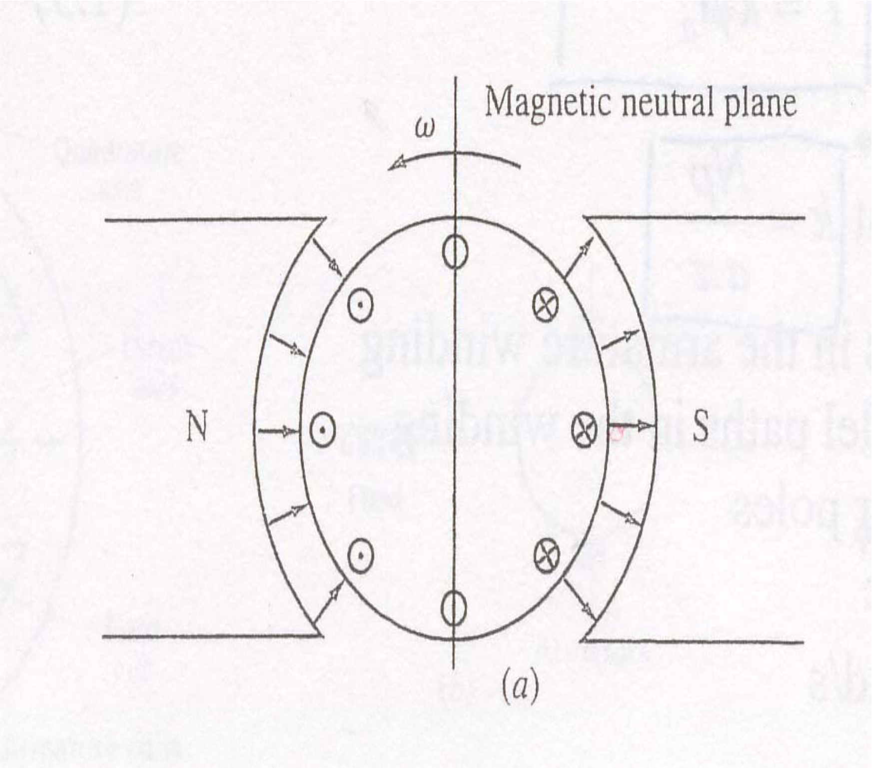


Figure 16.12 Commutation for a single armature winding.

Armature Reaction

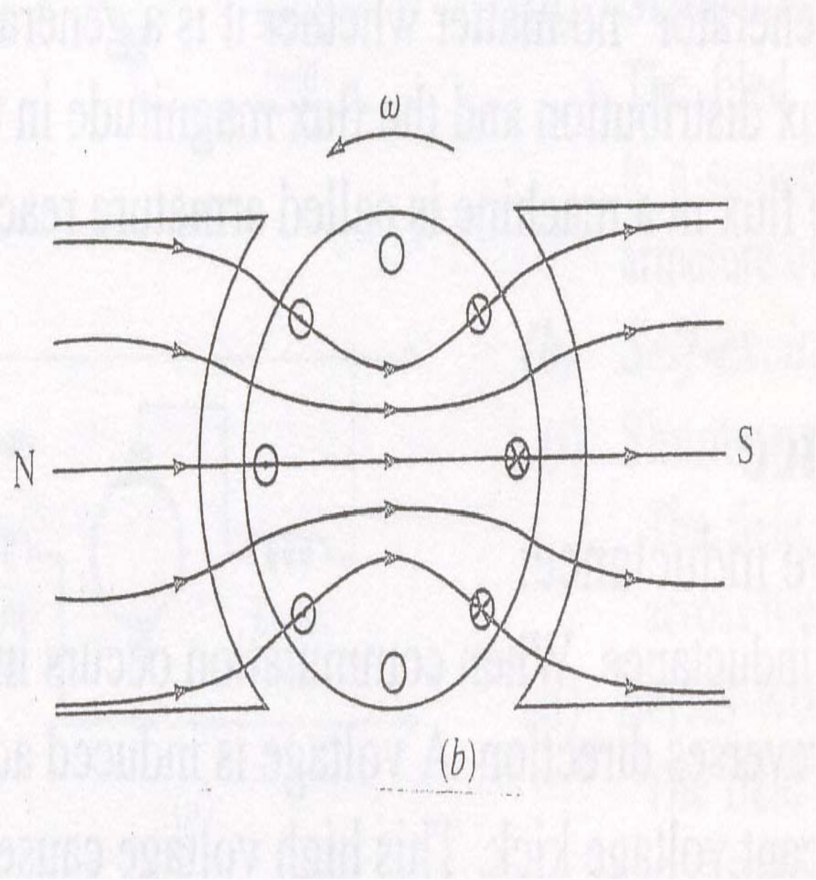
- Effect on flux distribution: *Neutral plane shift*
  - When current is flowing in the field winding, hence a flux is produced across the machine which flows from the North pole to the South pole.
  - Initially the pole flux is uniformly distributed and the magnetic neutral plane is vertical



(a)

Armature Reaction

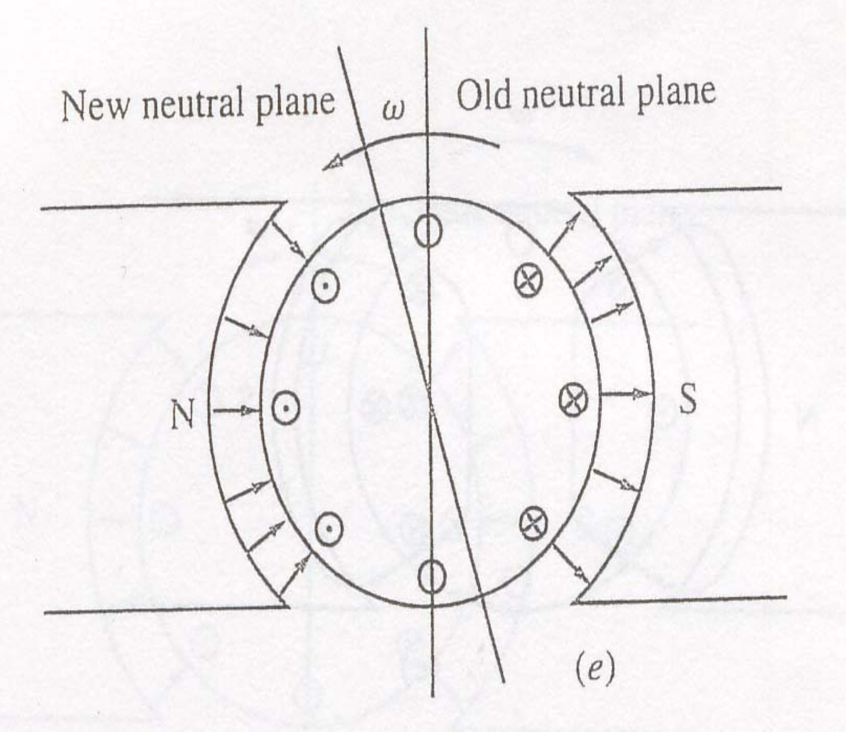
- Effect on flux distribution: *Neutral plane shift*
  - effect by the air gap on the flux field causes the distribution of flux is no longer uniform across the rotor.
  - There are two points on the periphery of the rotor where B= 0.



Armature Reaction

- Effect on flux distribution: *Neutral plane shift*

- The combined flux in the machine has the effect of strengthening or weakening the flux in the pole. Neutral axis is therefore shifted in the direction of motion.
- The result is current flow circulating between the shorted segments and large sparks at the brushes. The ending result is arcing and sparking at the brushes.



- **Solution to this problem:**
  - placing an additional poles on the neutral axis or mid-point that will produce flux density component, which counter-acts that produced by the armature.

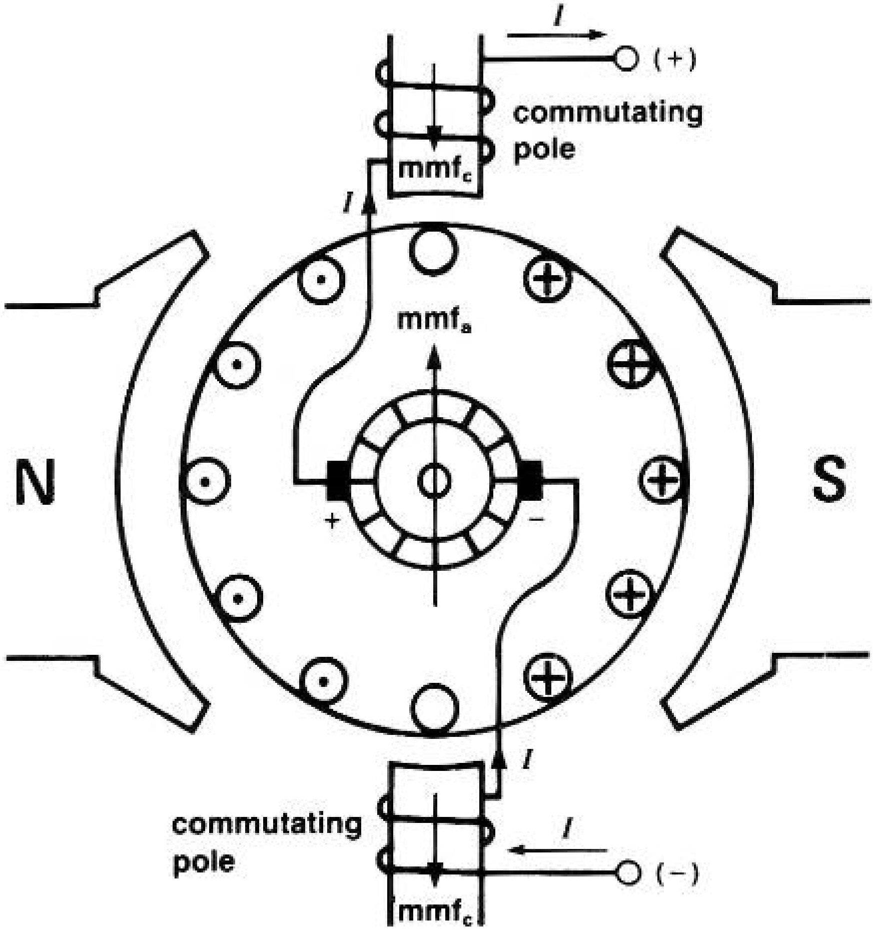


Figure 4.16
Commutating poles produce an mmf$_c$ that opposes the mmf$_a$ of the armature.

Schematic Connection Diagram of a DC Machine

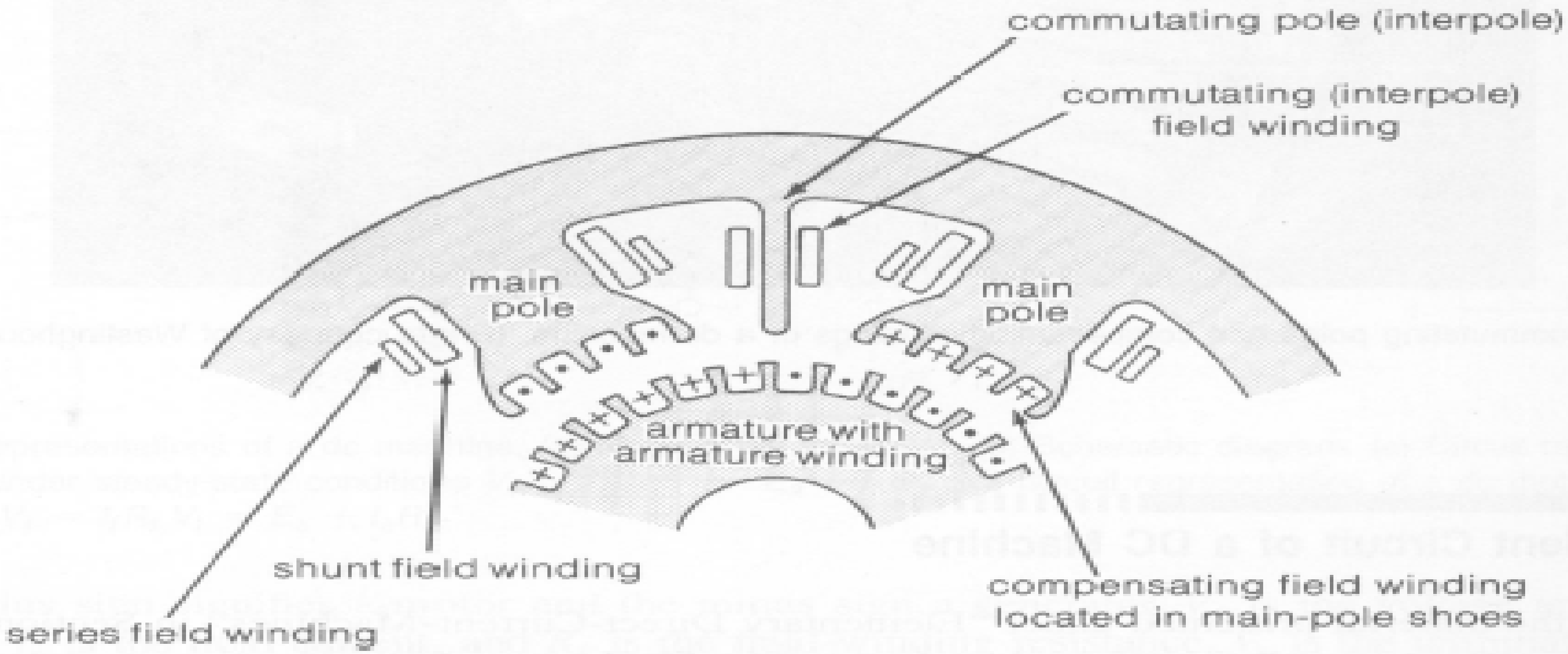


Figure 9.1.1 Section of a dc machine illustrating the arrangement of various field windings.

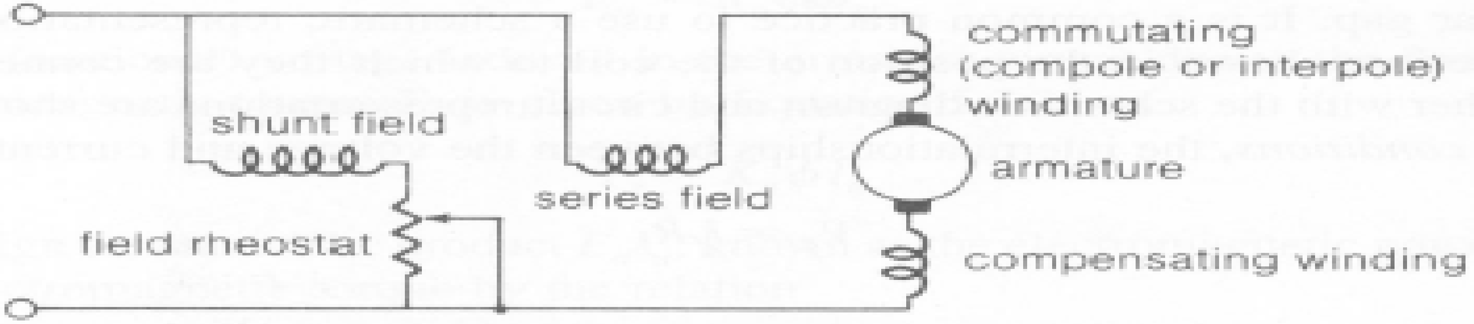
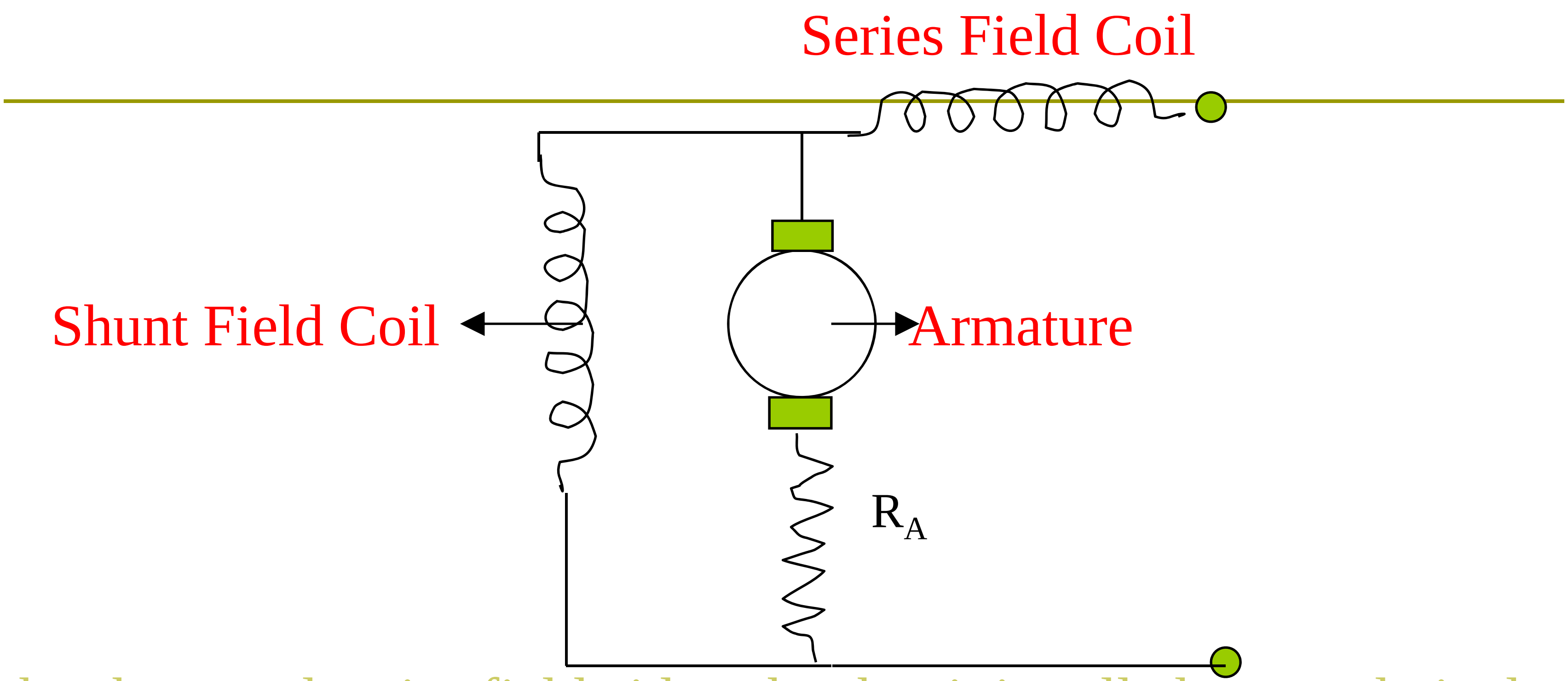


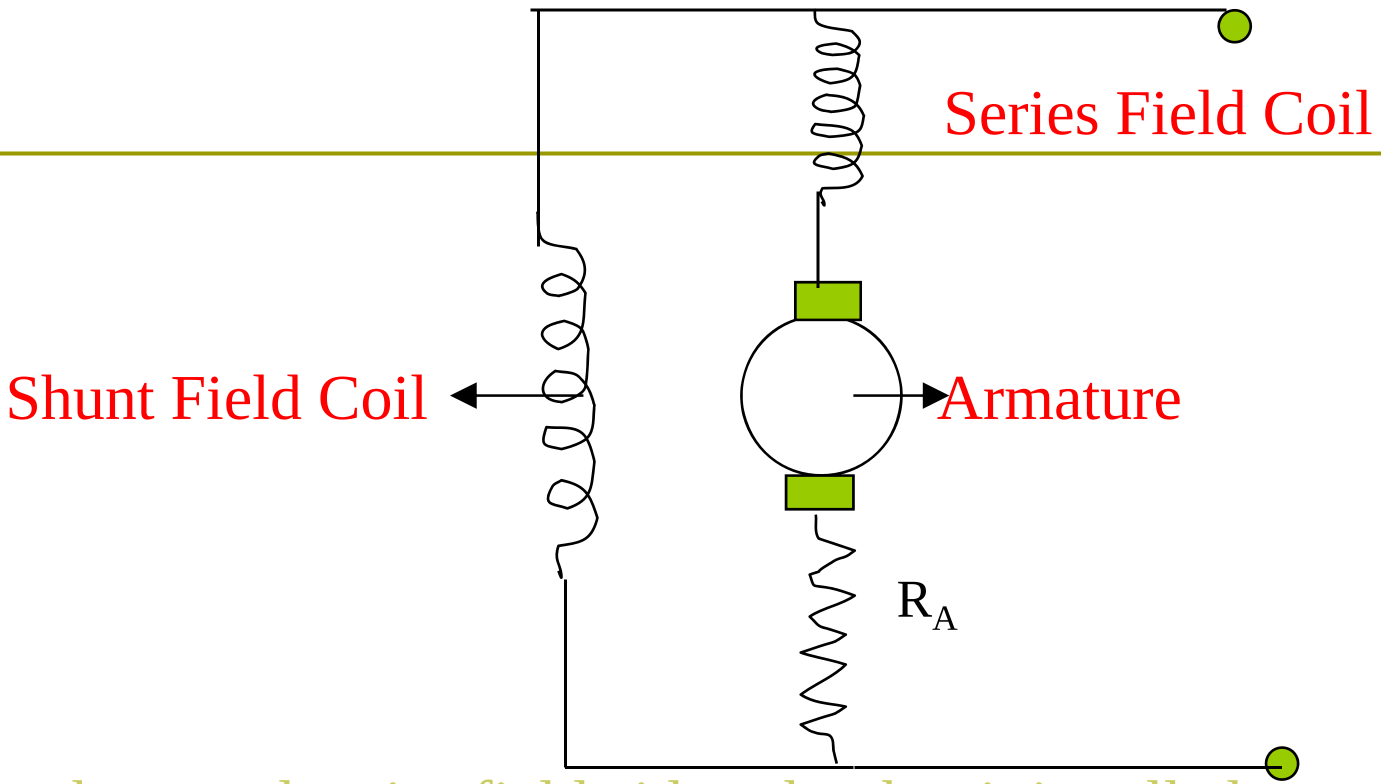
Figure 9.1.2 Schematic connection diagram of a dc machine.

Short shunt wound compound DC Machine



- If the shunt and series field aid each other it is called a cumulatively excited machine
- If the shunt and series field oppose each other it is called a differentially excited machine

Long shunt wound compound machine



- If the shunt and series field aid each other it is called a cumulatively excited machine
- If the shunt and series field oppose each other it is called a differentially excited machine

ROTOR WINDING:

- Rotor windings are of two types:
  - Lap winding
  - Wave winding

- No. of parallel paths:

  In lap winding p= no. of poles

  In wave winding p= 2

DC Generator Characteristics

In general, the following characteristics specify the steady-state performance of a DC generators:

1. Open-circuit characteristics: generated voltage versus field current at constant speed.
2. Load characteristic: terminal voltage versus load current at constant armature current and speed.

No Load Saturation curves

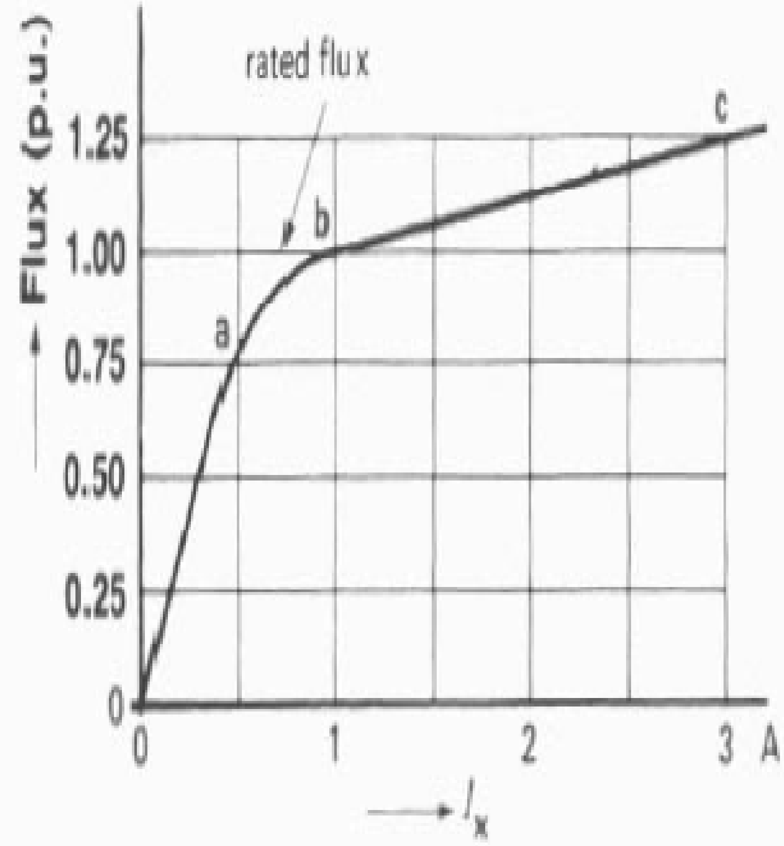


Figure 4.18a

Flux per pole versus exciting current.

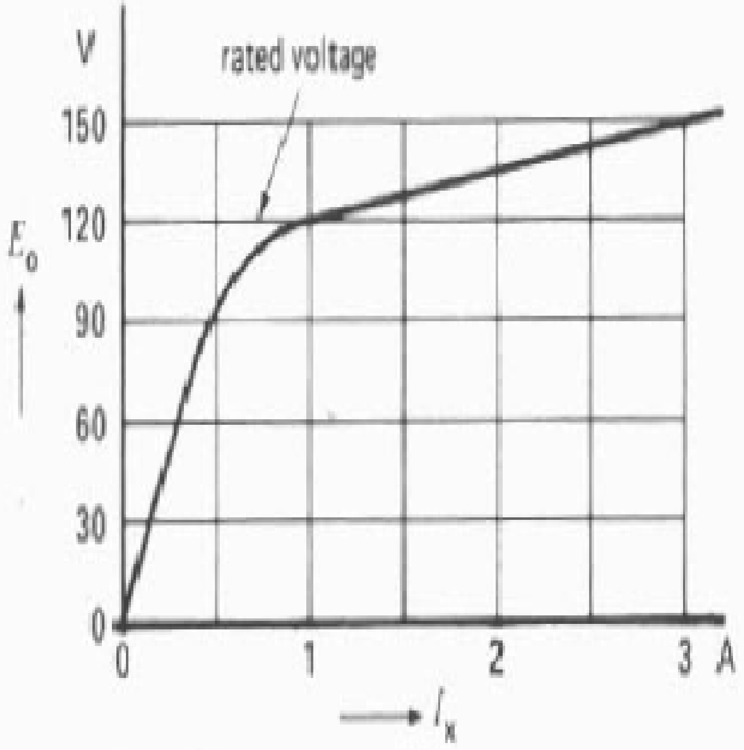


Figure 4.18b

Saturation curve of a dc generator.

Magnetic and Open circuit characteristics

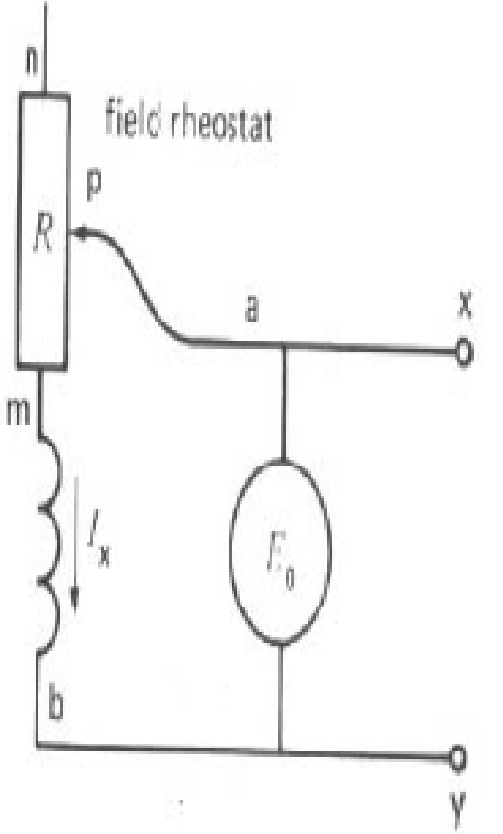


Figure 4.20

Controlling the generator voltage with a field rheostat. A rheostat is a resistor with an adjustable sliding contact.

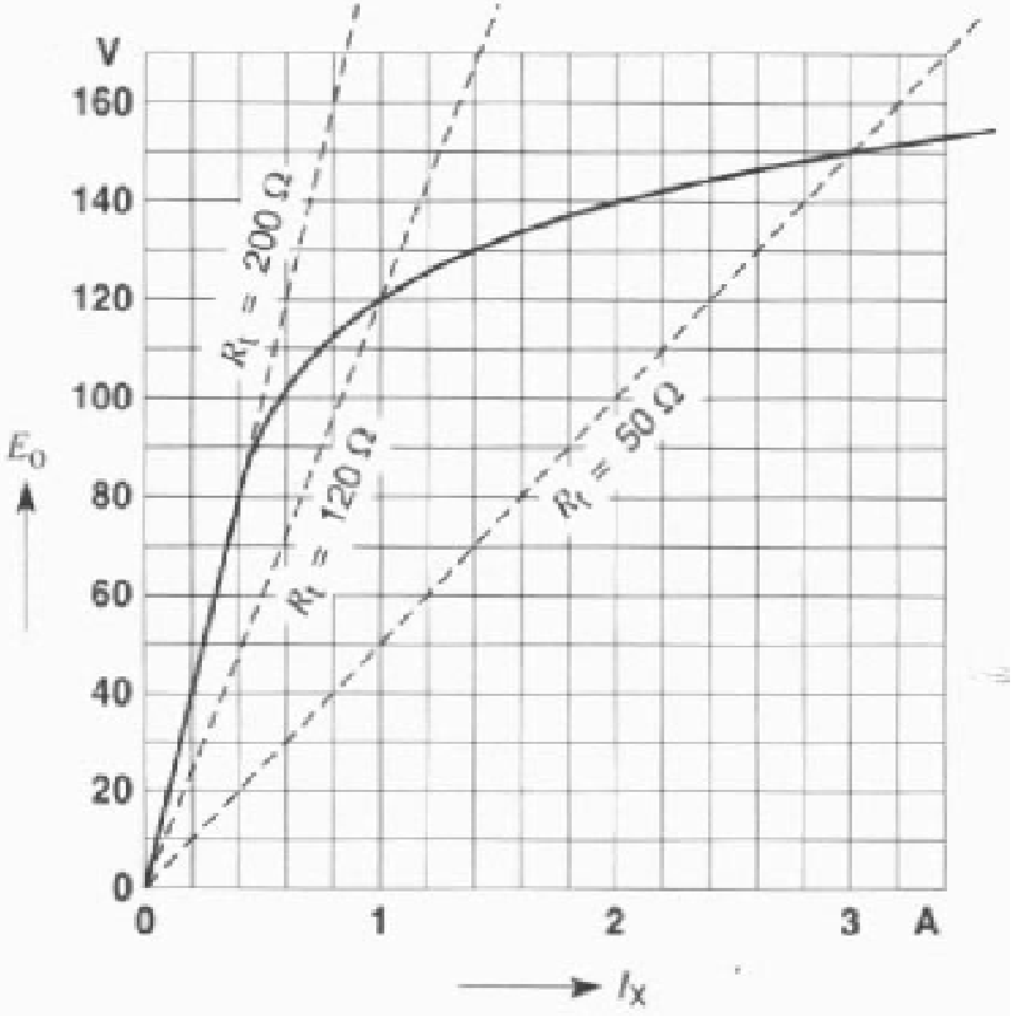
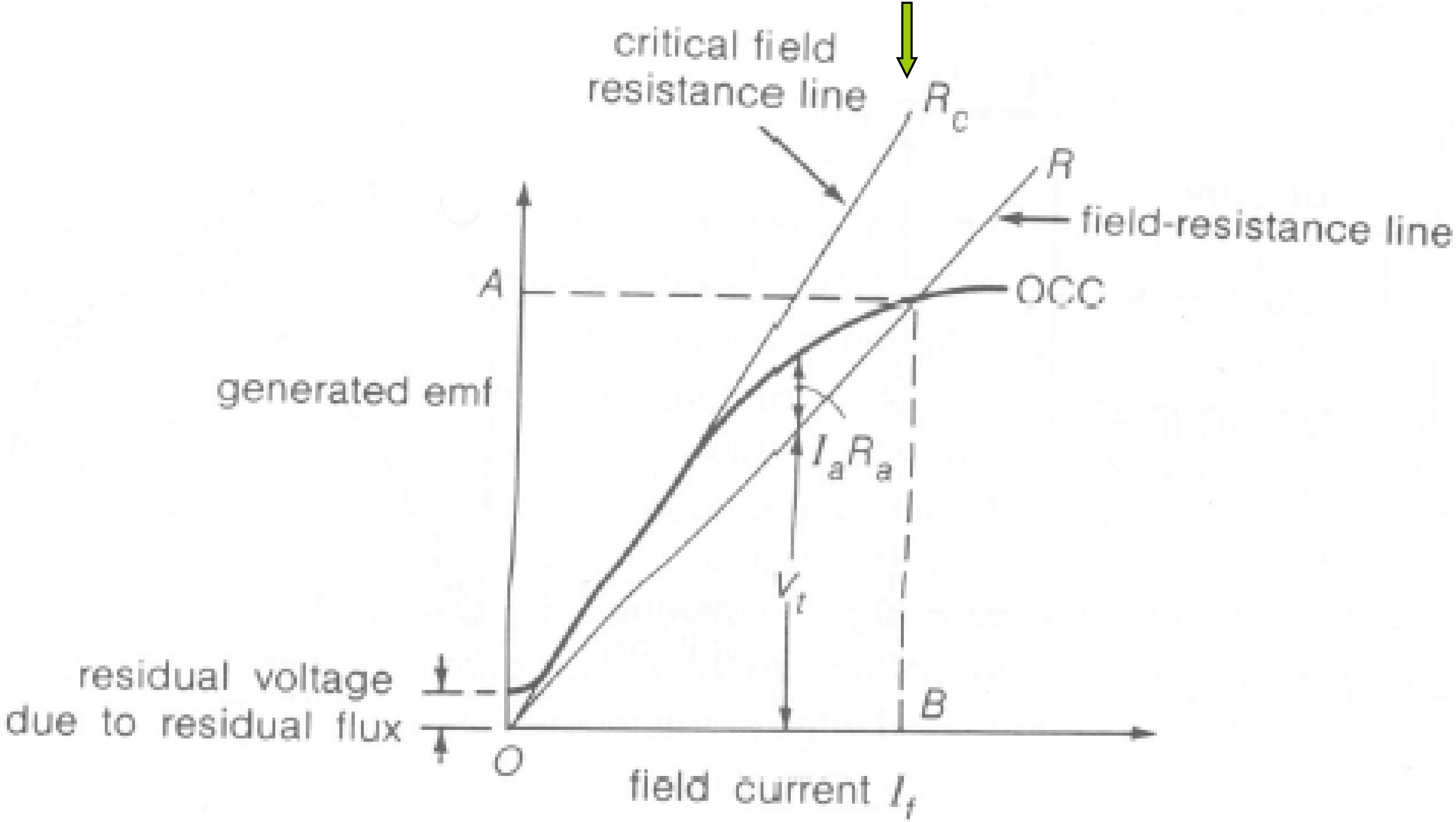


Figure 4.21

The no-load voltage depends upon the resistance of the shunt-field circuit.

Self-Excited DC Shunt Generator

Maximum permissible value of the field resistance if the terminal voltage has to build up.

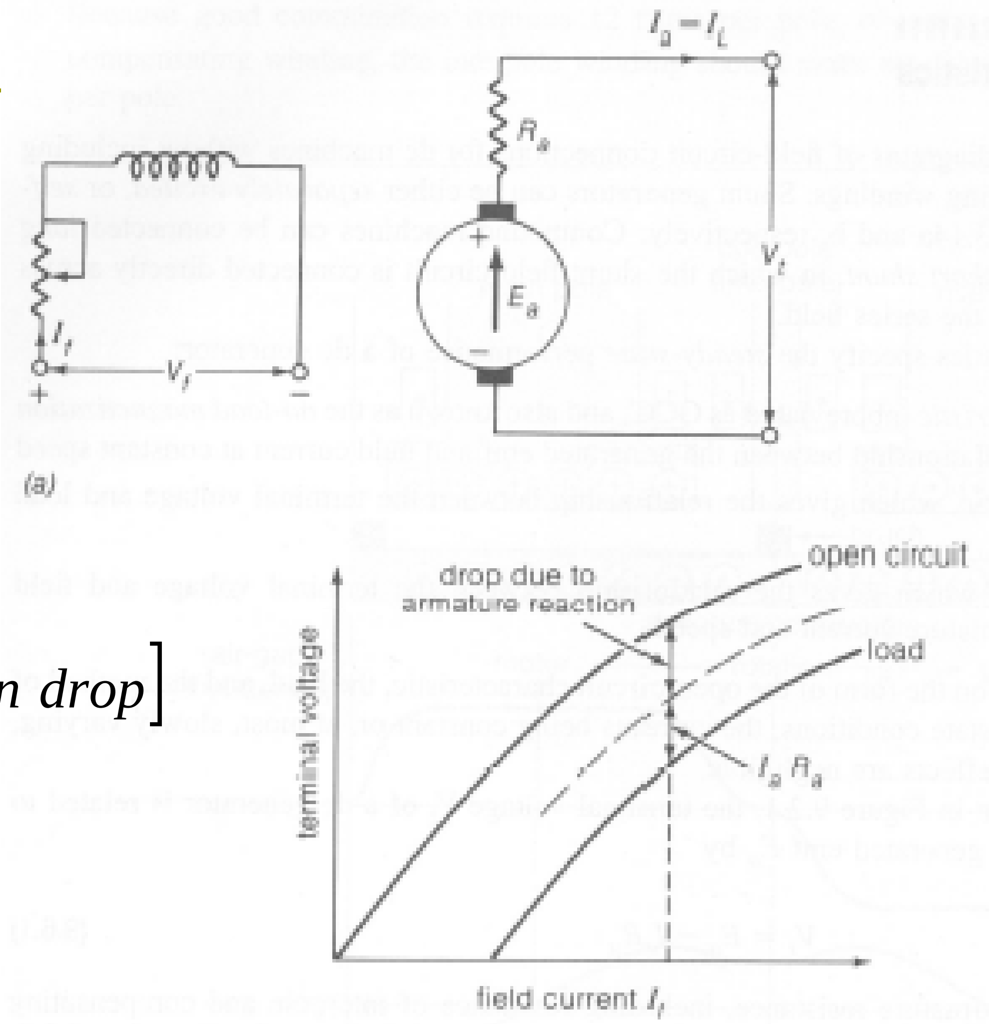


Open-circuit characteristic

DC Generator Characteristics

The terminal voltage of a dc generator is given by

$$
\begin{aligned}
V_t &= E_a - I_a R_a \\
&= \left[f\left(I_f, \omega_m\right) - Armature\ reaction\ drop \right] \\
&\quad - I_a R_a
\end{aligned}
$$



Open-circuit and load characteristics

Load characteristics of seperately excited generator

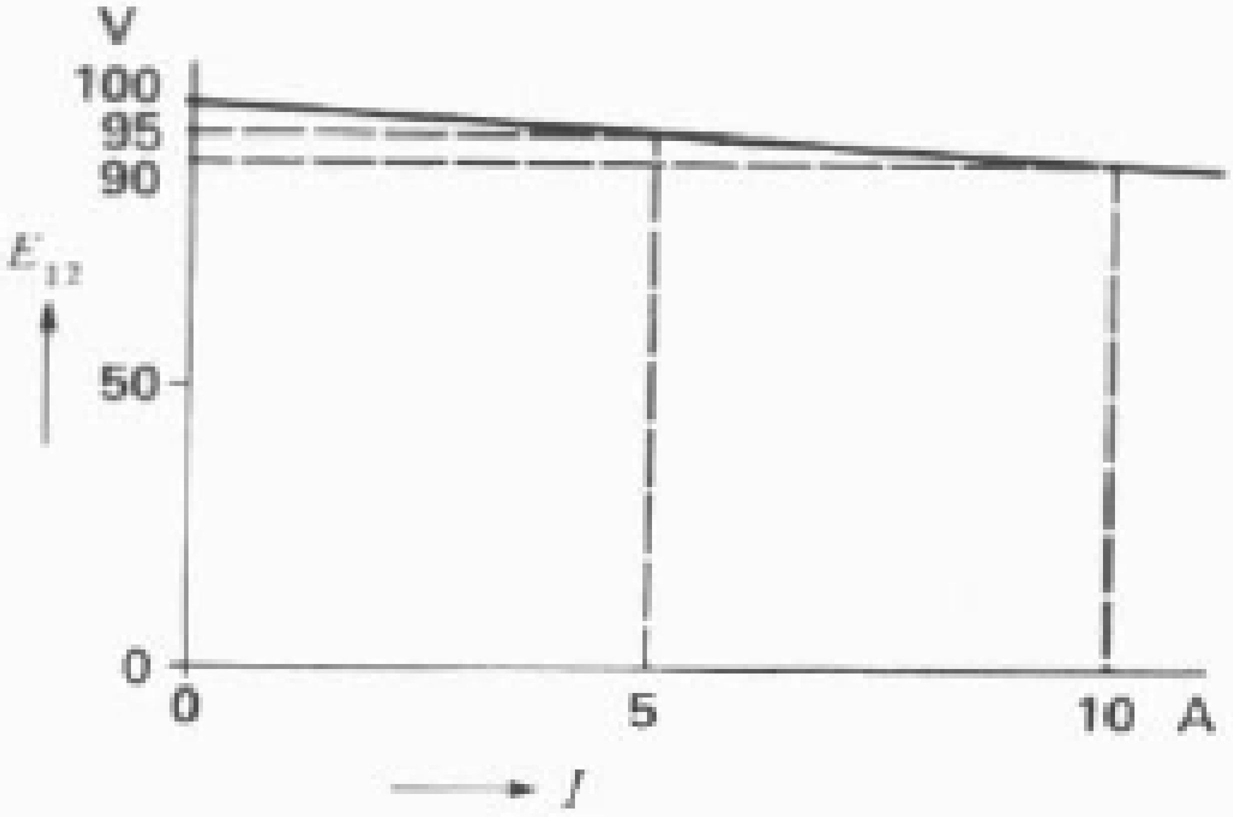


Figure 4.24
Load characteristic of a separately excited generator.

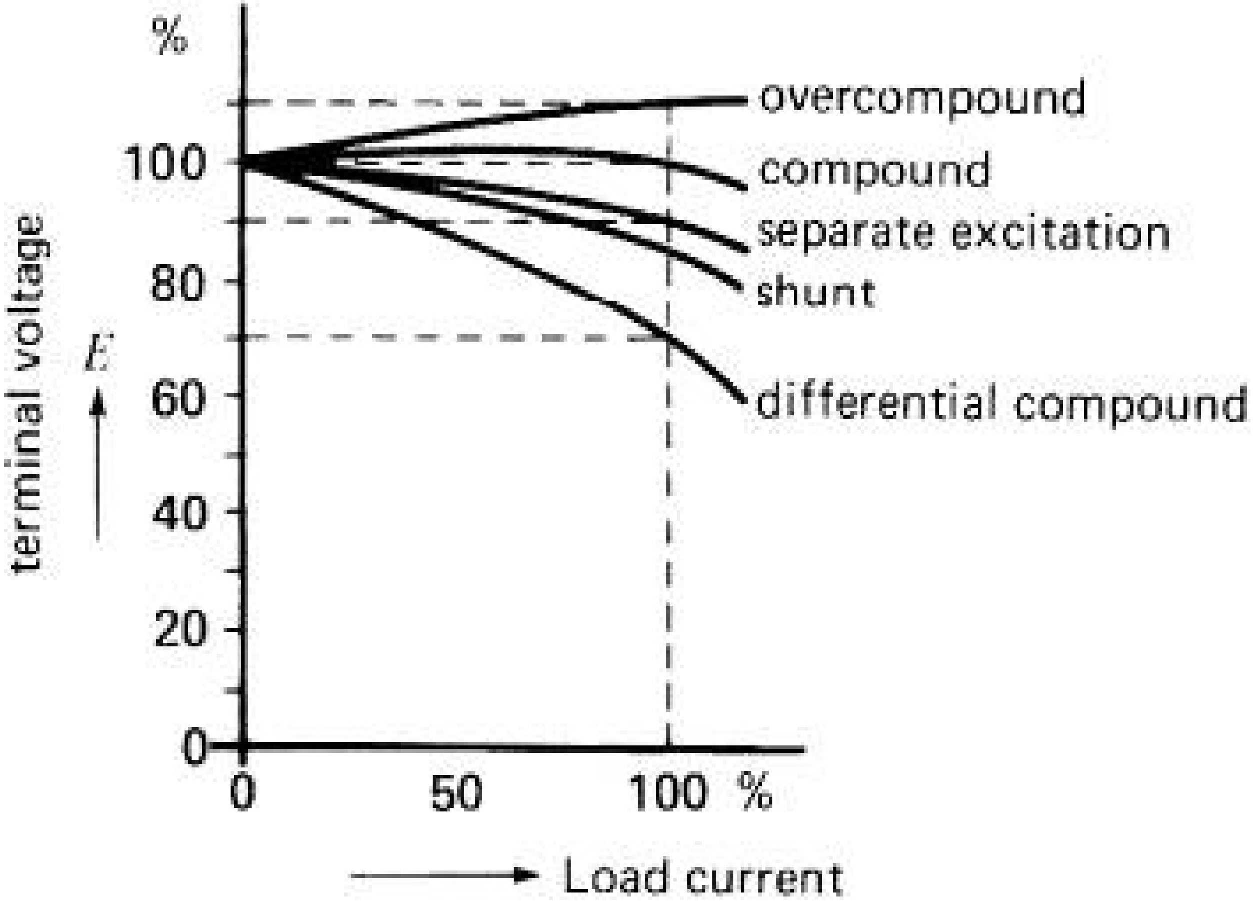
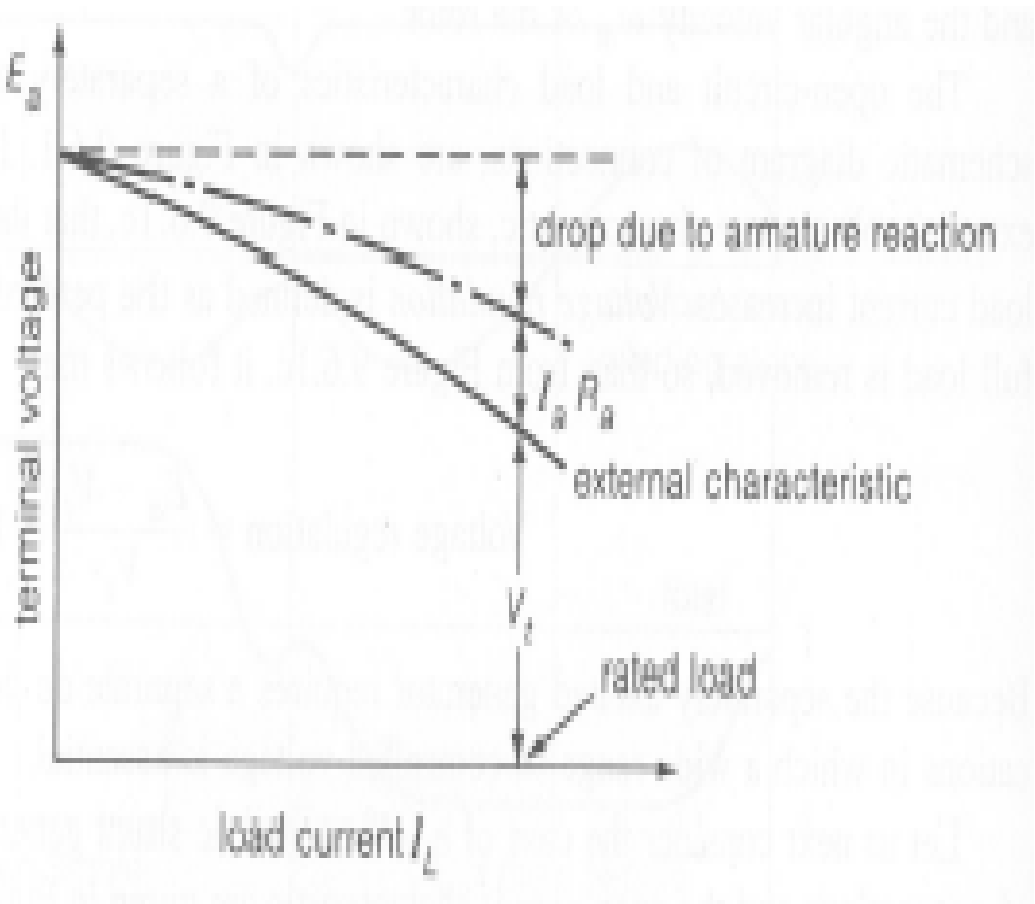


Figure 4.26
Typical load characteristics of dc generators.

DC Generator Characteristics

It can be seen from the external characteristics that the terminal voltage falls slightly as the load current increases.



External characteristics

Electromagnetic Torque

- Area per pole A $=2\pi rl/p$
- *FluX density* $=B/A=p\phi/2\pi rl \quad E_a I_a$
- Current / conductor is$=I_c= I_{a/A}$
- The force on a conductor is $F=BlI_a/A$
- The torque developed by a conductor=F.r

$$T=rBlI_a/A=p\phi I_a/2\pi a$$

The total torque developed is$=Zp\phi I_a/2\pi a=K\phi I_a$

$=E_a I_a/w_m$

DC MOTORS

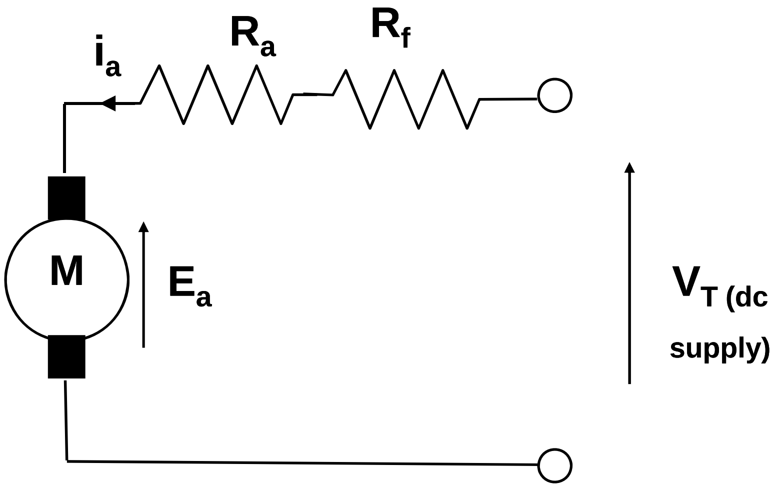
- DC motors consist of rotor-mounted windings (armature) and stationary windings (field poles). In all DC motors, except permanent magnet motors, current must be conducted to the armature windings by passing current through carbon brushes that slide over a set of copper surfaces called a commutator, which is mounted on the rotor.

Major types of dc motors

- Self excited dc motor
  - Series dc motor
  - Shunt dc motor
  - Compound dc motor
- Separately excited dc motor
- Permanent magnet dc motor

Series motors

- Series motors connect the field windings in series with the armature.
- Series motors lack good speed regulation, but are well-suited for high-torque loads like power tools and automobile starters because of their high torque production and compact size.



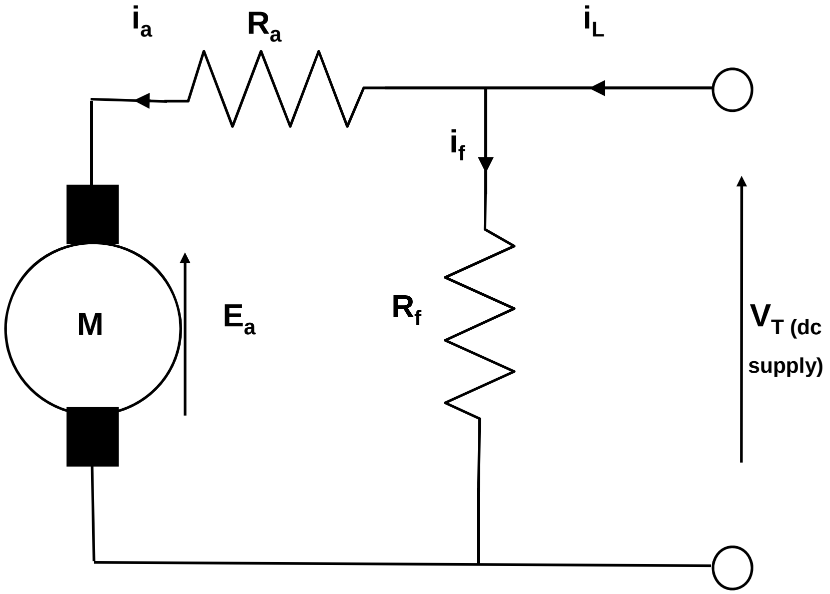
$$V_T = E_a + i_a(R_a + R_f)$$

$$note : i_a = i_L$$

$$E_a = K_1 K_2 I_a \varpi$$

Shunt motors

- Shunt motors use high-resistance field windings connected in parallel with the armature.
- Varying the field resistance changes the motor speed.
- Shunt motors are prone to armature reaction, a distortion and weakening of the flux generated by the poles that results in commutation problems evidenced by sparking at the brushes.
- Installing additional poles, called interpoles, on the stator between the main poles wired in series with the armature reduces armature reaction.



$$V_T = E_a + i_a(R_a)$$

$$note \quad : i_L = i_a + i_f$$

$$V_T = i_f R_f$$

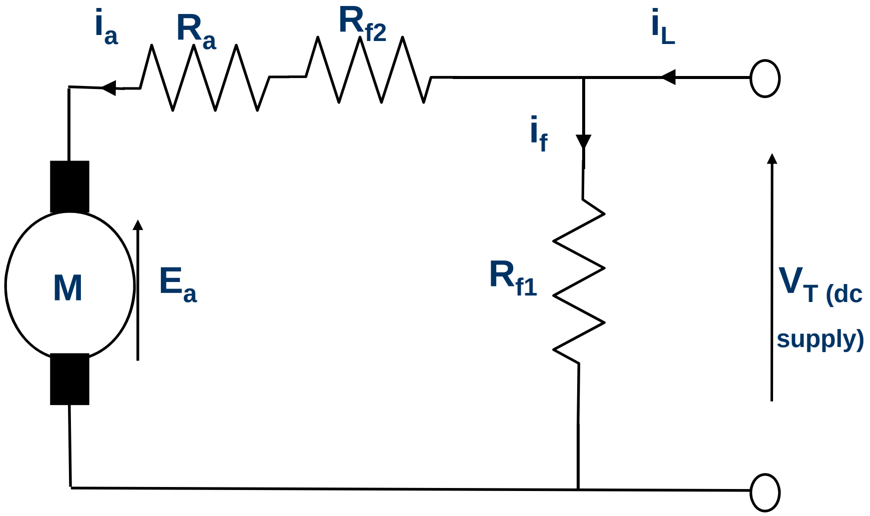
Compound motors

- the concept of the series and shunt designs are combined.

$$V_T = E_a + i_a(R_a + R_{f2})$$

$$note: i_L = i_a + i_f$$

$$V_T = i_f R_{f1}$$



Separately Excited Motor

- There is no direct connection between the armature and field winding resistance

- DC field current is supplied by an independent source
    - (such as battery or another generator or prime mover called an *exciter*)

Separately Excited Motor (Cont)

Circuit analysis:

$$E_a = \frac{NZp\Phi}{60A} = K_f i_f N = K_f N \Phi$$

Where p= no of poles

N= speed (rpm)

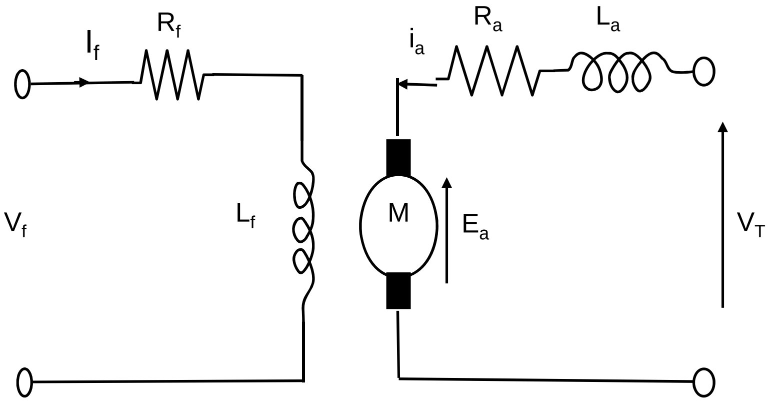
Z=no of conductor

Φ=Flux per pole (Wb)

A= no of current/parallel path

=p (lap winding)

=2 (wave winding)



KVL:

$$V_f = i_f R_f$$

$$V_T = E_a + i_a R_a$$

$$note : i_a = i_L$$

Speed control of Dc motors

$$E_a = \frac{NZp\Phi}{60A} = K_f i_f N = K_f N\Phi$$

$$V_T = E_a + i_a R_a$$

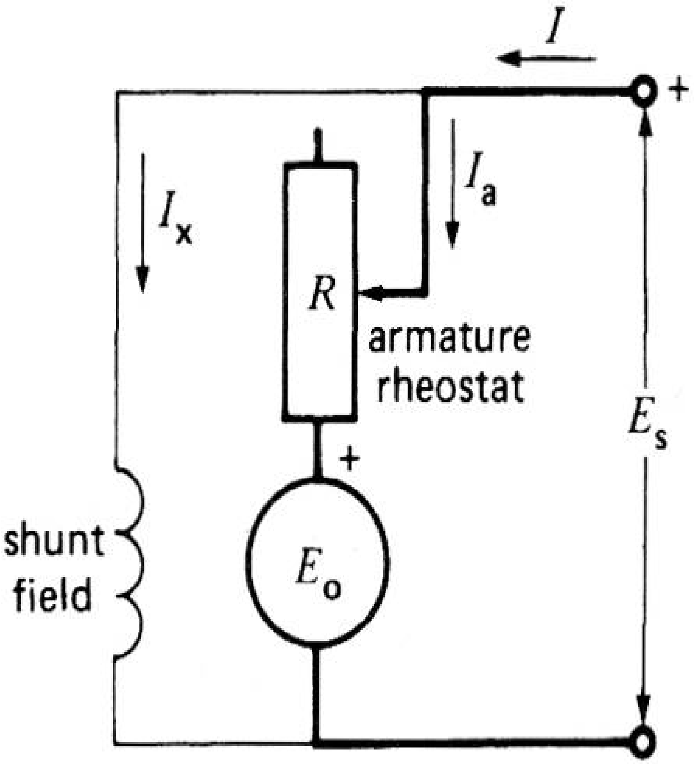
$$N = \frac{V_T - i_a R_a}{K\phi}$$

Speed control by various methods

- Field flux
- Armature control
- Terminal voltage

$$V_T = E_a + i_a R_a$$

$$N = \frac{V_T - i_a R_a}{K \phi}$$



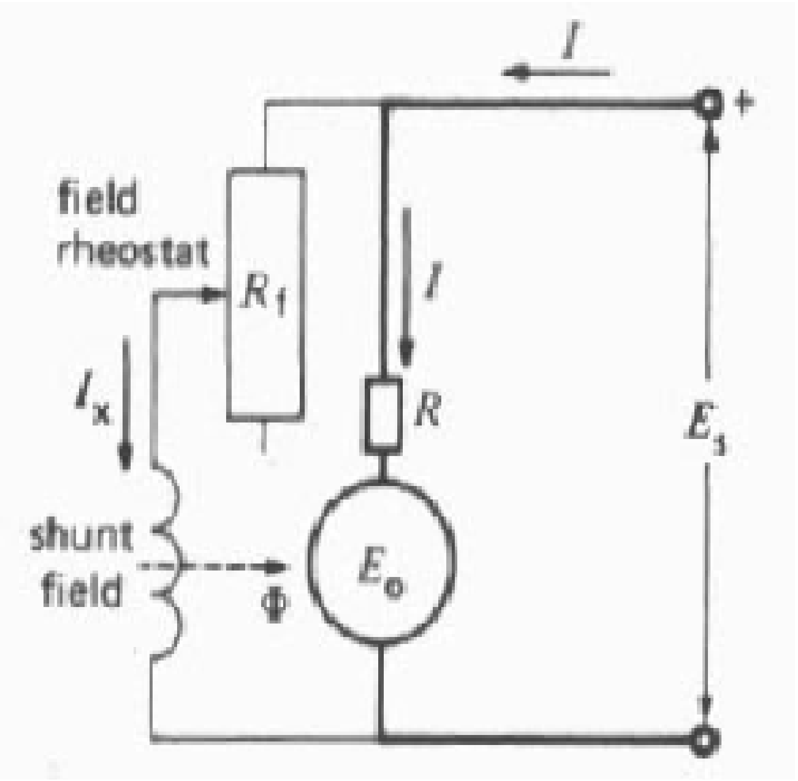


Figure 5.7

Armature speed control using a rheostat.

The expressions for motor speed show that the speed of the motor is directly proportional to the armature supply voltage and inversely proportional to the flux per pole. This gives rise to two methods of controlling the speed of DC motors:

Speed Control for shunt motor and separately excited dc motor

i. **Armature resistance speed control**

- Speed may be controlled by changing R_a
- The total resistance of armature may be varied by means of a rheostat in series with the armature
- The armature speed control rheostat also serves as a starting resistor.

Speed Control for shunt motor and separately excited dc motor

- Advantages armature resistance speed control:
  1. Starting and speed control functions may be combined in one rheostat
  2. The speed range begins at zero speed
  3. The cost is much less than other system that permit control down to zero speed
  4. Simple method

- Disadvantages armature resistance speed control :
  1. Introduce more power loss in rheostat
  2. Speed regulation is poor (S.R difference n_{Loaded} & $n_{no\ loaded}$)
  3. Low efficiency due to rheostat

Speed Control for shunt motor and separately excited dc motor

ii. **Field Speed Control**

- Rheostat in series with field winding (shunt or separately etc.)
- If field current, I_f is varied, hence flux is also varied
- Not suitable for series field

Speed Control for shunt motor and separately excited dc motor

- Advantages field speed control:
  1. Allows for controlling at or above the base speed
  2. The cost of the rheostat is cheaper because I_f is small value

- Disadvantages field speed control :
  1. Speed regulation is poor (S.R difference n_{Loaded} & $n_{no\ loaded}$)
  2. At high speed, flux is small, thus causes the speed of the machines becomes unstable
  3. At high speed also, the machines is unstable mechanically, thus there is an upper speed limit

Speed Control for shunt motor and separately excited dc motor

- Advantages armature terminal voltage speed control:
  1. Does not change the speed regulation
  2. Speed is easily controlled from zero to maximum safe speed

- Disadvantages armature terminal voltage speed control :
  1. Cost is higher because of using power electronic controller

Speed Control in Shunt DC Motors

Field Control:

R_a and V_t are kept constant, field rheostat is varied to change the field current.

$$N = \frac{Vt - IaRa}{K\phi}$$

For no-load condition, T_e=0. So, no-load speed varies inversely with the field current.

Speed control from zero to base speed is usually obtained by armature voltage control. Speed control beyond the base speed is obtained by decreasing the field current. If armature current is not to exceed its rated value (heating limit), speed control beyond the base speed is restricted to constant power, known as constant power application.

$$P = V_t I_a = const = E_a I_a = T_e \omega_m$$

$$T_e = \frac{E_a I_a}{\omega_m} = \frac{const.}{\omega_m}$$

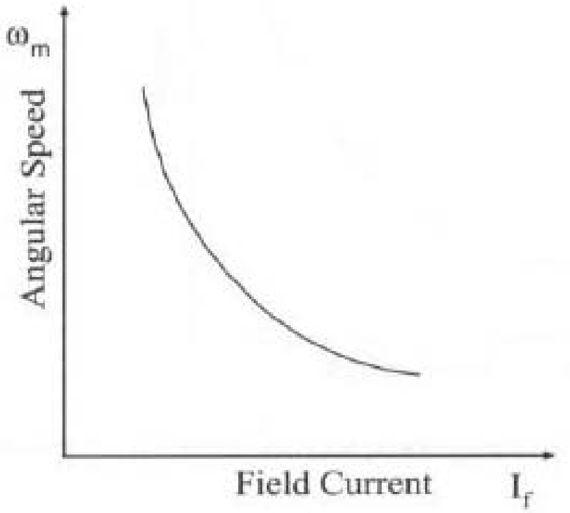


Figure 2: No-load speed vs. field current

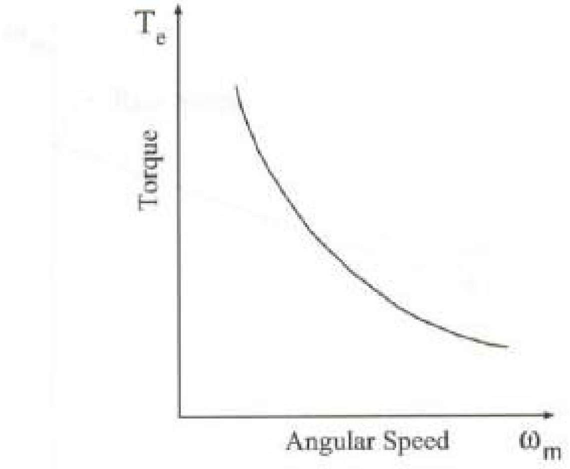


Figure 3a: I_f control, constant power operation

Speed Control in Shunt DC Motors

Armature Voltage Control:

R_a and I_f are kept constant and the armature terminal voltage is varied to change the motor speed.

$$\omega_m = K_1 V_t - K_2 T_e$$

$$K_1 = \frac{1}{K_a \phi_d}; \quad K_2 = \frac{1}{(K_a \phi_d)^2} \quad ;\phi_d \text{ is const.}$$

For constant load torque, such as applied by an elevator or hoist crane load, the speed will change linearly with V_t. In an actual application, when the speed is changed by varying the terminal voltage, the armature current is kept constant. This method can also be applied to series motor.

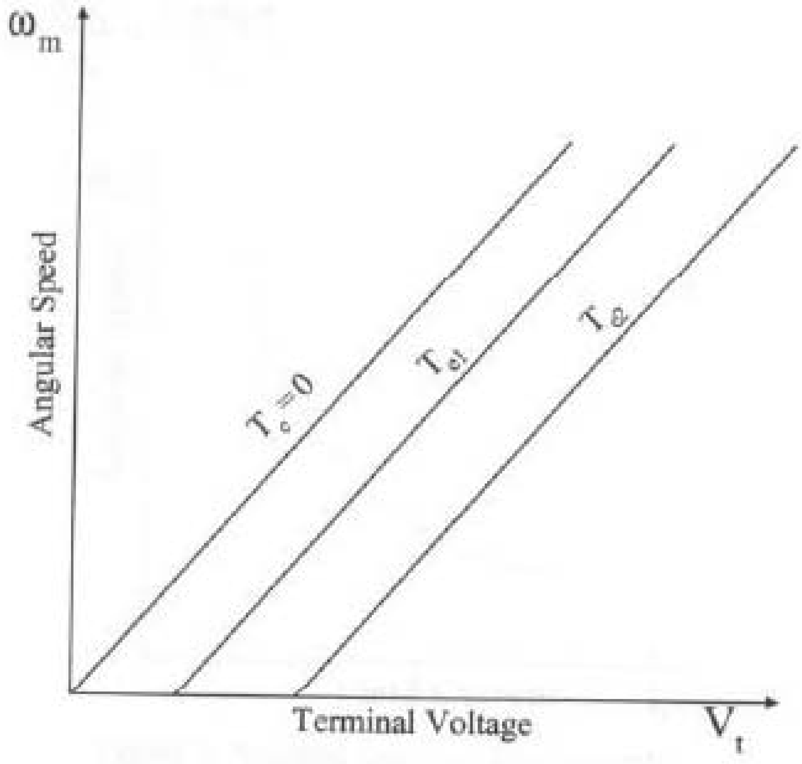


Figure 1: Speed vs. terminal voltage

Speed Control in Shunt DC Motors

Armature Resistance Control:

V_t *and* I_f are kept constant at their rated value, armature resistance is varied.

$$\omega_m = \frac{V_t}{K_a\phi_d} - \frac{R_a + R_{adj}}{(K_a\phi_d)^2} T_e = K_5 - K_6 T_e$$

The value of R_{adj} can be adjusted to obtain various speed such that the armature current I_a (hence torque, $T_e = K_a\phi_d I_a$) remains constant.

Armature resistance control is simple to implement. However, this method is less efficient because of loss in R_{adj}. This resistance should also been designed to carry armature current. It is therefore more expensive than the rheostat used in the field control method.

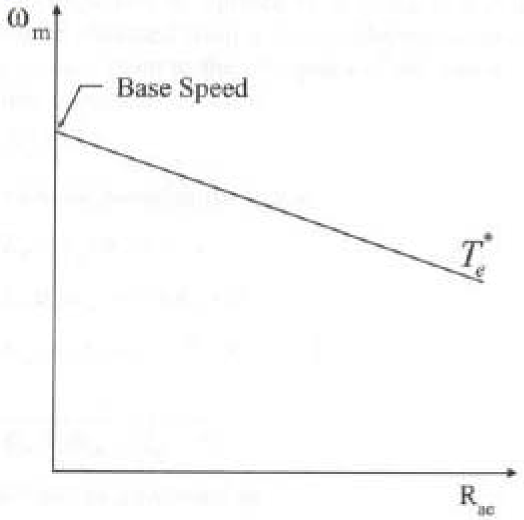


Figure 4: Angular speed vs. armature resistance for a torque of T_e^*

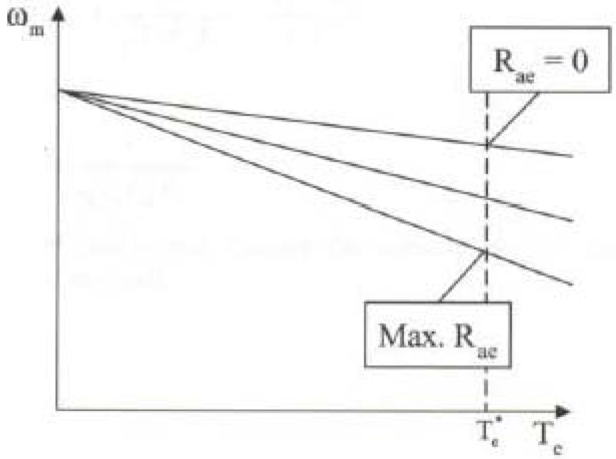


Figure 5: Speed-torque characteristics at different armature resistance

Speed Control in Series DC Motors

Armature Voltage Control:

A variable dc voltage can be applied to a series motor to control its speed. A variable dc voltage can be obtained from a power electronic converter.

$$\phi_d = K_s I_a$$

$$\begin{aligned} V_t &= E_a + I_a(R_a + R_s) \\ &= K_a \phi_d \omega_m + I_a(R_a + R_s) \\ &= K_a(K_s I_a)\omega_m + I_a(R_a + R_s) \end{aligned}$$

$$I_a = \frac{V_t}{K_a K_s \omega_m + R_a + R_s}$$

Torque in a series motor can be expressed as

$$\begin{aligned} T_e &= K_a \phi_d I_a = K_a K_s I_a^2 \\ &= \frac{K_a K_s V_t^2}{\left[K_a K_s \omega_m + (R_a + R_s)\right]^2} \end{aligned}$$

$$or,\ \omega_m = \frac{V_t}{\sqrt{T_e K_a K_s}} - \frac{R_a + R_s}{K_a K_s} \approx \frac{V_t}{\sqrt{T_e K_a K_s}}$$

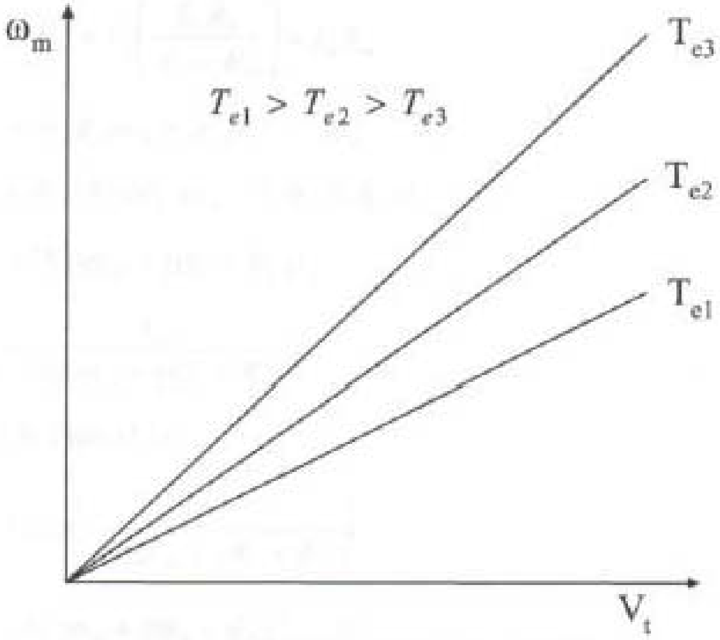


Figure 6: Speed vs. armature voltage

Speed Control in Series DC Motors

Field Control:

Control of field flux in a sries motor is achieved by using a diverter resistance.
The developed torque can be expressed as.

$$T_e = K_a \phi_d I_a = K_a K_s \left(\frac{R_d}{R_s + R_d} \right) I_a^2 = K\rho I_a^2$$

$$where,\ K = K_a K_s\ and\ \rho = \frac{R_d}{R_s + R_d}$$

$$\begin{aligned} V_t &= E_a + \left(\frac{R_s R_d}{R_s + R_d} \right) I_a + I_a R_a \\ &= K_a \phi_d \omega_m + \rho I_a R_s + I_a R_a \\ &= K_a (K_s \rho I_a) \omega_m + (\rho R_s + R_a) I_a \\ &= (K\rho\omega_m + \rho R_s + R_a) I_a \end{aligned}$$

$$or,\ I_a = \frac{V_t}{K\rho\omega_m + \rho R_s + R_a}$$

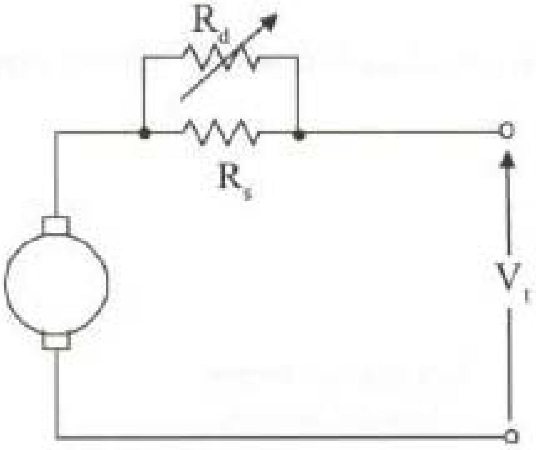


Figure 7: A series motor with a field diverter resistance

Speed Control in Series DC Motors

$$T_e = K\rho\left(\frac{V_t}{K\rho\omega_m + \rho R_s + R_a}\right)^2$$

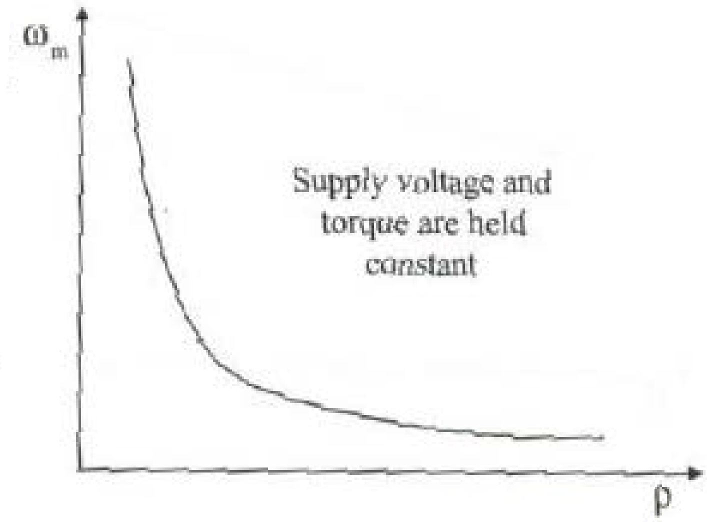


Figure 8: Speed vs. ?

Speed Control in Series DC Motors

Armature Resistance Control:

Torque in this case can be expressed as

$$T_e = \frac{KV_t^2}{\left(R_a + R_{adj} + R_s + K\omega_m\right)^2}$$

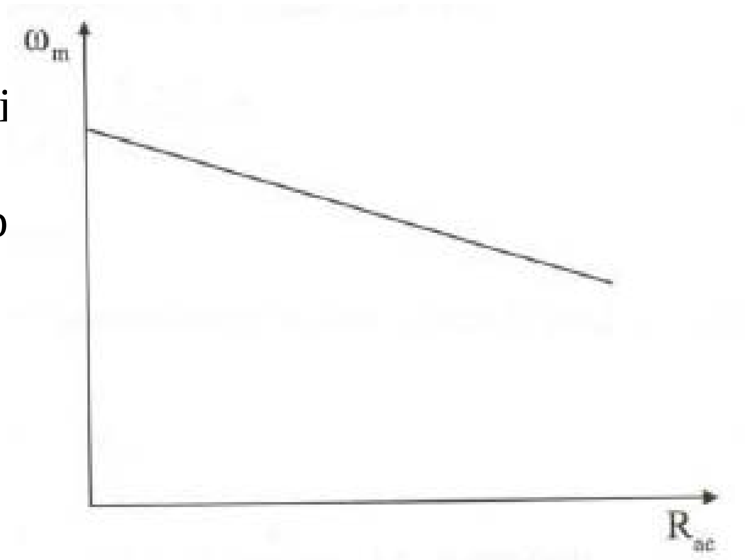


Figure 9: Speed vs. armature resistance

R_{ae} is an external resistance connected in series wi the armature.

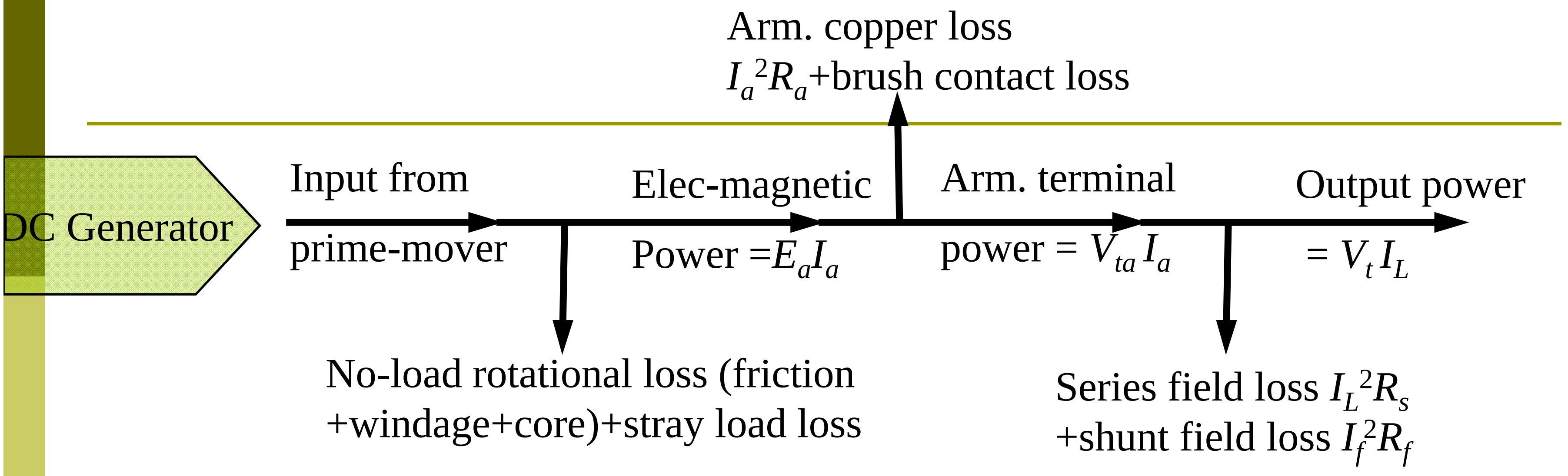
For a given supply voltage and a constant develop torque, the term (R_a+R_{ae}+R_s+$K\omega_m$) should remain constant. Therefore, an increase in R_{ae} must be accompanied by a corresponding decrease in ω_m.

$$\left(R_a + R_{adj} + R_s + K\omega_m\right)^2 = \frac{KV_t^2}{T_e}$$

$$or,\ R_a + R_{adj} + R_s + K\omega_m = \sqrt{\frac{K}{T_e}}V_t$$

$$or,\ \omega_m = \frac{V_t}{\sqrt{KT_e}} - \frac{R_a + R_{adj} + R_s}{K}$$

Power Division in DC Machines



DC Generator

Input from prime-mover

Elec-magnetic Power $=E_aI_a$

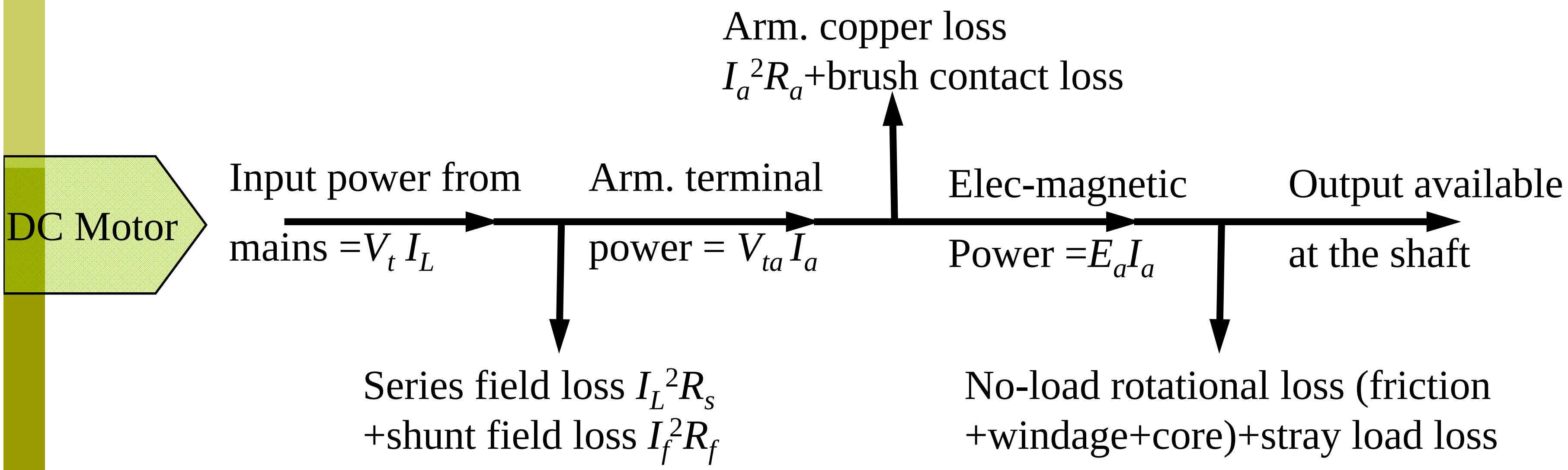
Arm. copper loss $I_a^2R_a$+brush contact loss

Arm. terminal power $= V_{ta}\,I_a$

Output power $= V_t\,I_L$

No-load rotational loss (friction +windage+core)+stray load loss

Series field loss $I_L^2R_s$ +shunt field loss $I_f^2R_f$



DC Motor

Input power from mains $=V_t\,I_L$

Arm. terminal power $= V_{ta}\,I_a$

Arm. copper loss $I_a^2R_a$+brush contact loss

Elec-magnetic Power $=E_aI_a$

Output available at the shaft

Series field loss $I_L^2R_s$ +shunt field loss $I_f^2R_f$

No-load rotational loss (friction +windage+core)+stray load loss

Efficiency

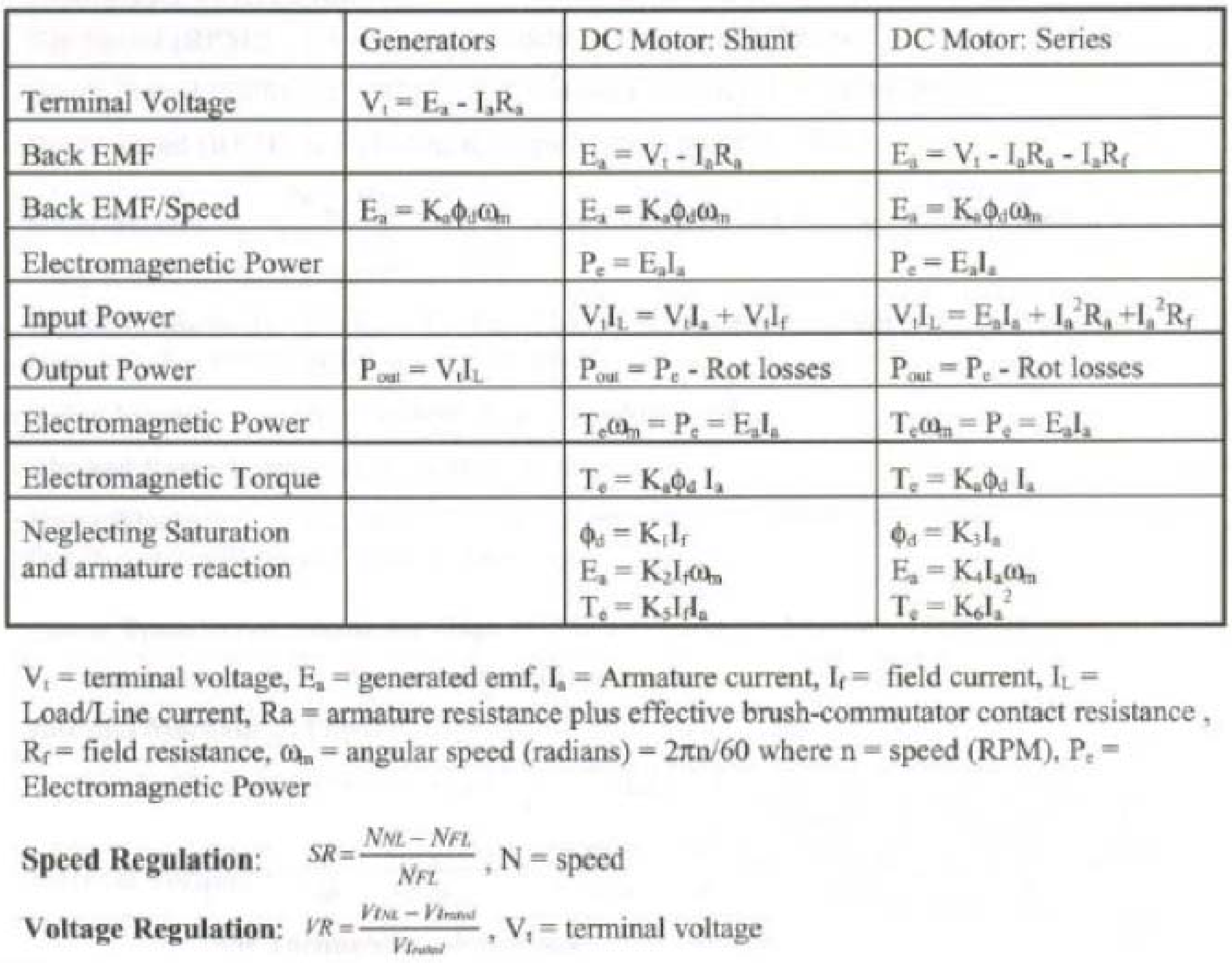
$$\eta = \frac{Power\ \ Output}{Power\ \ Input}$$

$$= \frac{Power\ \ Input - Losses}{Power\ \ Input}$$

$$= 1 - \frac{Losses}{Power\ \ Input}$$

The losses are made up of rotational losses (3-15%), armature circuit copper losses (3-6%), and shunt field copper loss (1-5%). The voltage drop between the brush and commutator is 2V and the brush contact loss is therefore calculated as $2I_a$.

DC Machines Formulas



	Generators	DC Motor: Shunt	DC Motor: Series
Terminal Voltage	$V_t = E_a - I_aR_a$		
Back EMF		$E_a = V_t - I_aR_a$	$E_a = V_t - I_aR_a - I_aR_f$
Back EMF/Speed	$E_a = K_a\phi_d\omega_m$	$E_a = K_a\phi_d\omega_m$	$E_a = K_a\phi_d\omega_m$
Electromagenetic Power		$P_e = E_aI_a$	$P_e = E_aI_a$
Input Power		$V_tI_L = V_tI_a + V_tI_f$	$V_tI_L = E_aI_a + I_a^2R_a + I_a^2R_f$
Output Power	$P_{out} = V_tI_L$	$P_{out} = P_e$ - Rot losses	$P_{out} = P_e$ - Rot losses
Electromagnetic Power		$T_e\omega_m = P_e = E_aI_a$	$T_e\omega_m = P_e = E_aI_a$
Electromagnetic Torque		$T_e = K_a\phi_d I_a$	$T_e = K_a\phi_d I_a$
Neglecting Saturation and armature reaction		$\phi_d = K_1I_f$ $E_a = K_2I_f\omega_m$ $T_e = K_5I_fI_a$	$\phi_d = K_3I_a$ $E_a = K_4I_a\omega_m$ $T_e = K_6I_a^2$

V_t = terminal voltage, E_a = generated emf, I_a = Armature current, I_f = field current, I_L = Load/Line current, Ra = armature resistance plus effective brush-commutator contact resistance , R_f = field resistance, ω_m = angular speed (radians) = $2\pi n/60$ where n = speed (RPM), P_e = Electromagnetic Power

Speed Regulation: $SR = \frac{N_{NL} - N_{FL}}{N_{FL}}$, N = speed

Voltage Regulation: $VR = \frac{V_{tNL} - V_{trated}}{V_{trated}}$, V_t = terminal voltage