Digital Signal Processing- Lecture 13

Topics to be covered:

- Impulse Invariant Method

Impulse Invariant Method

Indeed, in the general case the poles are mapped as

$$\alpha_k \rightarrow e^{\alpha_k T}$$

since any rational transfer function with the numerator degree strictly less than the denominator degree can be decomposed to partial fractions

$$H_c(s) = \sum \frac{C}{s - \alpha_k}$$

and similarly it can be shown

$$H(z) = \sum \frac{C}{1 - e^{\alpha_k T} z^{-1}}$$

Impulse Invariant Method: Stability

Since poles are mapped as:

$$\alpha_k \rightarrow e^{\alpha_k T}$$

stable analogue filter is transformed into stable digital filter

$$s = \sigma + j\Omega \leftrightarrow z = re^{j\omega}$$

$$\sigma < 0 \Rightarrow \left|e^{\alpha_k T}\right| < 1$$

Summary of the Impulse Invariant Method

- Determine analogue filter $H_c(s)$ satisfying specifications for desired digital filter (not discussed here!).
- If necessary, expand $H_c(s)$ using partial fractions.
- Obtain the z-transform of each partial fraction z
- Obtain $H(z)$ by combining the z-transforms of the partial fractions.

Summary of the Impulse Invariant Method

- Advantage:
  - preserves the order and stability of the analogue filter
- Disadvantages:
  - Not applicable to all filter types (high-pass, band-stop)
  - There is distortion of the shape of frequency response due to aliasing

Matched z-transform method

- **Matched z-transform:** very simple method to convert analog filters into digital filters.

$$H(s) = \frac{\prod_{k=1}^{M}(s - z_k)}{\prod_{k=1}^{N}(s - p_k)} \overset{\text{matched z-transform}}{\rightarrow} H(z) = \frac{\prod_{k=1}^{M}\left(1 - e^{z_k T} z^{-1}\right)}{\prod_{k=1}^{N}\left(1 - e^{p_k T} z^{-1}\right)};$$

i.e. poles and zeros are transformed according to

$$z_k \rightarrow e^{z_k T}, \; p_k \rightarrow e^{p_k T}$$

where T is the sampling period.

- Poles using this method are similar to impulse invariant method.
- Zeros are located at a new position.

Backward Difference Method

The analogue-domain variable *s* represents differentiation.

We can try to replace s by approximating differentiation operator in the digital domain:

$$\left.\frac{dx(t)}{dt}\right|_{t=nT} = \frac{x(nT) - x((n-1)T)}{T} = \frac{x(n) - x(n-1)}{T}$$

Thus,

$$y(t) = \frac{dx(t)}{dt} \Rightarrow y(n) \approx \frac{x(n) - x(n-1)}{T}$$

$$Y(z) \approx T^{-1}\left(1 - z^{-1}\right) X(z)$$

Which suggests the *s*-to-*z* transformation:

$$s \leftarrow T^{-1}\left(1 - z^{-1}\right)$$