Compiler Design

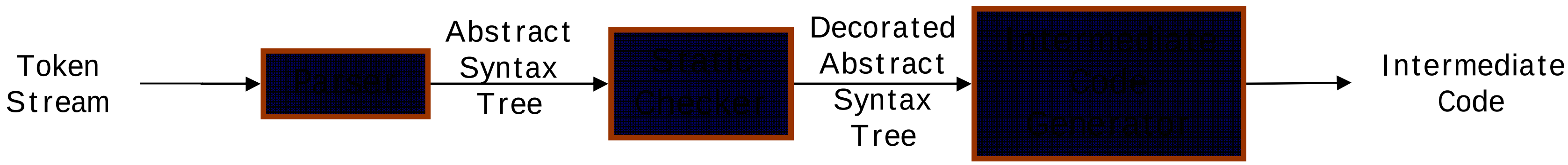
Lecture-21

Type Checking

Topics Covered

- Type Checking
- Type Expressions
- Equivalence of Type Expressions
- Overloading Functions & Operators

Static Checking



- Static (Semantic) Checks
  - Type checks: operator applied to incompatible operands?
  - Flow of control checks: break (outside while?)
  - Uniqueness checks: labels in case statements
  - Name related checks: same name?

Type Checking

- Problem: Verify that a type of a construct matches that expected by its context.

- Examples:
  - mod requires integer operands (PASCAL)
  - * (dereferencing) – applied to a pointer
  - a[i] – indexing applied to an array
  - f(a1, a2, …, an) – function applied to correct arguments.

- Information gathered by a type checker:
  - Needed during code generation.

Type Systems

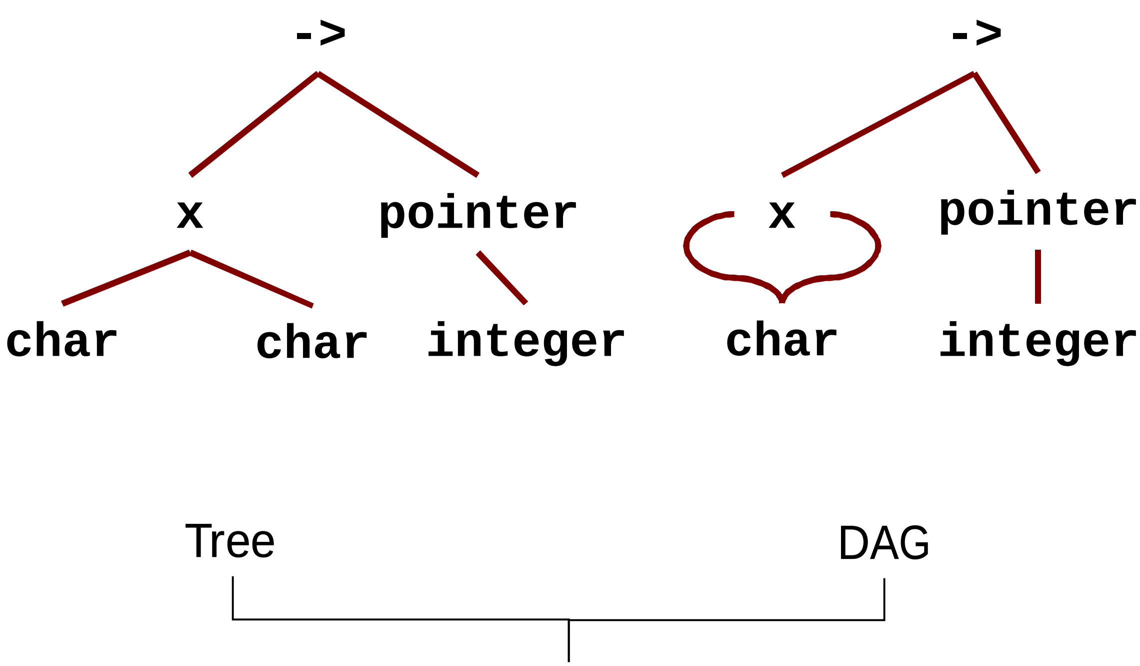
- A collection of rules for assigning type expressions to the various parts of a program.
- Based on: Syntactic constructs, notion of a type.
- Example: If both operators of “+”, “-”, “*” are of type integer then so is the result.
- Type Checker: An implementation of a type system.
  - Syntax Directed.
- Sound Type System: eliminates the need for checking type errors during run time.

Type Expressions

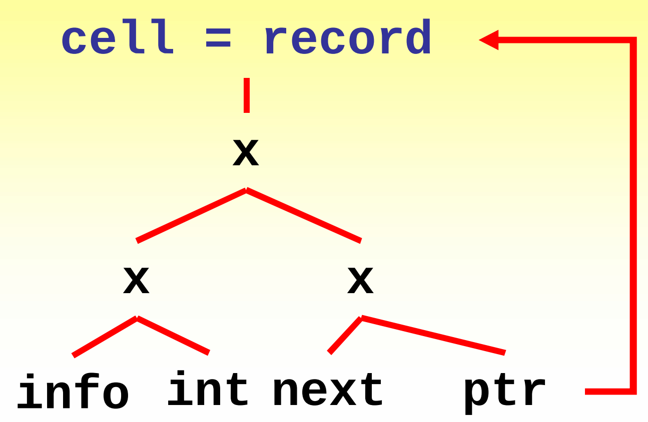
- Implicit Assumptions:
  - Each program has a type
    - → Expressions
    - → Statements
  - Types have a structure

Basic Types		Type Constructors
Boolean	Character	Arrays
Real	Integer	Records
Enumerations	Sub-ranges	Sets
Void	Error	Pointers
Variables	Names	Functions

Representation of Type Expressions



(char x char)-> pointer (integer)




```
struct cell {
        int info;
        struct cell * next;
};
```

Type Expressions Grammar

```
Type ->    int | float | char | …
           | void
           | error
           | name
           | variable
           | array( size, Type)
           | record( (name, Type)*)
           | pointer( Type)
           | tuple((Type)*)
           | arrow(Type, Type)
```

- Basic Types: int, float, char, …, void, error, name, variable
- Structured Types: array, record, pointer, tuple, arrow

A Simple Typed Language

Program -> Declaration; Statement
Declaration -> Declaration; Declaration
| id: Type
Statement -> Statement; Statement
| id := Expression
| if Expression then Statement
| while Expression do Statement
Expression -> literal | num | id
| Expression mod Expression
| E[E] | E ↑ | E (E)

Type Checking Expressions

E -> int_const

E -> float_const

E -> id

E -> E1 + E2

Type Checking Expressions

E -> E1 [E2]

E -> *E1

E -> &E1

E -> E1 (E2)

E -> (E1, E2)

Type Checking Statements

S -> id := E

S -> if E then S1

S -> while E do S1

S -> S1; S2

Equivalence of Type Expressions

Problem: When in E1.type = E2.type?

- We need a precise definition for type equivalence
- Interaction between type equivalence and type representation

Example:

```
type vector = array [1..10] of real
type weight = array [1..10] of real
var x, y: vector; z: weight
```

Name Equivalence: When they have the same name.

- x, y have the same type; z has a different type.

Structural Equivalence: When they have the same structure.

- x, y, z have the same type.

Structural Equivalence

- Definition: by Induction
  - Same basic type (basis)
  - Same constructor applied to SE Type (induction step)
  - Same DAG Representation
- In Practice: modifications are needed
  - Do not include array bounds – when they are passed as parameters
  - Other applied representations (More compact)
- Can be applied to: Tree/ DAG
  - Does not check for cycles
  - Later improve it.

Algorithm Testing Structural Equivalence

```
function stequiv(s, t): boolean
{
    if (s & t are of the same basic type) return true;

    if (s = array(s1, s2) & t = array(t1, t2))
            return equal(s1, t1) & stequiv(s2, t2);

    if (s = tuple(s1, s2) & t = tuple(t1, t2))
            return stequiv(s1, t1) & stequiv(s2, t2);

    if (s = arrow(s1, s2) & t = arrow(t1, t2))
            return stequiv(s1, t1) & stequiv(s2, t2);

    if (s = pointer(s1) & t = pointer(t1))
            return stequiv(s1, t1);
}
```

Recursive Types

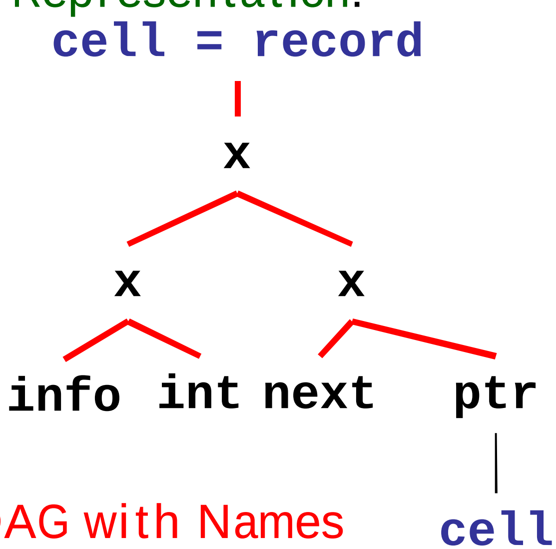
Where: Linked Lists, Trees, etc.

How: records containing pointers to similar records

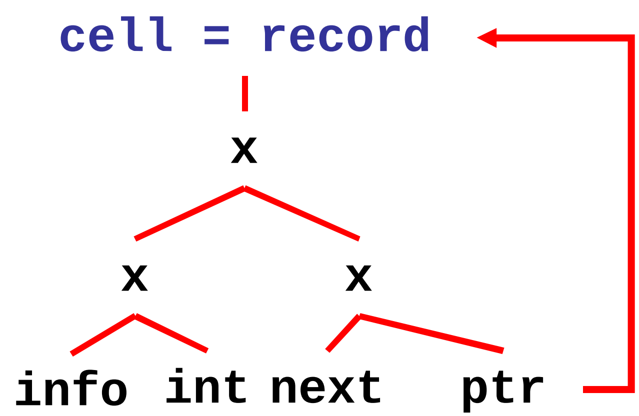
Example: type link = ↑ cell;

cell = record info: int; next = link end

Representation:



DAG with Names



Substituting names out (cycles)

Recursive Types in C

- C Policy: avoid cycles in type graphs by:
  - Using structural equivalence for all types
  - Except for records -> name equivalence
- Example:
  - **`struct cell {int info; struct cell * next;}`**
- Name use: name cell becomes part of the type of the record.
  - Use the acyclic representation
  - Names declared before use – except for pointers to records.
  - Cycles – potential due to pointers in records
  - Testing for structural equivalence stops when a record constructor is reached ~ same named record type?

Overloading Functions & Operators

- Overloaded Symbol: one that has different meanings depending on its context
- Example: Addition operator +
- Resolving (operator identification): overloading is resolved when a unique meaning is determined.
- Context: it is not always possible to resolve overloading by looking only the arguments of a function
  - Set of possible types
  - Context (inherited attribute) necessary

Overloading Example

```
function "*" (i, j: integer) return complex;
function "*" (x, y: complex) return complex;
* Has the following types:
    arrow(tuple(integer, integer), integer)
    arrow(tuple(integer, integer), complex)
    arrow(tuple(complex, complex), complex)
int i, j;
k = i * j;
```

Narrowing Down Types

E' -> E

{E'.types = E. types
E.unique = if E'.types = {t} then t else error}

E -> id

{E.types = lookup(id.entry)}

E -> E1(E2)

{E.types = {s' | ∃ s ∈ E2.types and S->s' ∈ E1.types}
t = E.unique
S = {s | s ∈ E2.types and S->t ∈E1.types}
E2.unique = if S ={s} the S else error
E1.unique = if S = {s} the S->t else error

Polymorphic Functions

- Defn: a piece of code (functions, operators) that can be executed with arguments of different types.
- Examples: Built in Operator indexing arrays, pointer manipulation
- Why use them: facilitate manipulation of data structures regardless of types.
- Example HL:

```
fun length(lptr) = if null (lptr) then 0
                   else length(+l(lptr)) + 1
```

A Language for Polymorphic Functions

P -> D ; E

D -> D ; D | id : Q

Q -> ∀ α. Q | T

T -> arrow (T, T) | tuple (T, T)

| unary (T) | (T)

| basic

| α

E -> E (E) | E, E | id

Type Variables

- Why: variables representing type expressions allow us to talk about unknown types.
  - Use Greek alphabets α, β, γ …
- Application: check consistent usage of identifiers in a language that does not require identifiers to be declared before usage.
  - A type variable represents the type of an undeclared identifier.
- Type Inference Problem: Determine the type of a language constant from the way it is used.
  - We have to deal with expressions containing variables.

Examples of Type Inference

```
Type link ↑ cell;
Procedure mlist (lptr: link; procedure p);
{ while lptr <> null { p(lptr); lptr := lptr
  ↑ .next} }
```

Hence: p: link -> void

```
Function deref (p)
{ return p ↑; }
```

P: β, β = pointer(α)

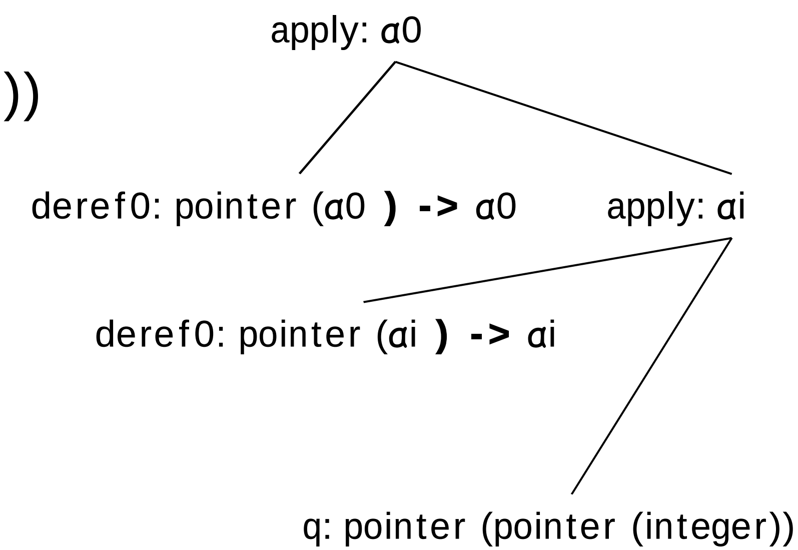
Hence deref: ∀ α. pointer(α) -> α

Program in Polymorphic Language

deref: ∀ α. pointer(α) -> α

q: pointer (pointer (integer))

deref (deref((q))



Notation:

-> arrow

x tuple

Subsripts i and o distinguish between the inner and outer occurrences of deref, respectively.

Type Checking Polymorphic Functions

- Distinct occurrences of a p.f. in the same expression need not have arguments of the same type.
  - deref (deref (q))
  - Replace α with fresh variable and remove ∀ (αi, αo)

- The notion of type equivalence changes in the presence of variables.
  - Use unification: check if s and t can be made structurally equivalent by replacing type vars by the type expression.

- We need a mechanism for recording the effect of unifying two expressions.
  - A type variable may occur in several type expressions.

Substitutions and Unification

- Substitution: a mapping from type variables to type expressions.

Function subst (t: type Expr): type Expr { S
    if (t is a basic type) return t;
    if (t is a basic variable) return S(t); --identify if t $\notin$ S
    if (t is t1 -> t2) return subst(t1) -> subst (t2); }

- Instance: S(t) is an instance of t written S(t) < t.
    - Examples: pointer (integer) < pointer (α) , int -> real $\neq$ α-> α

- Unify: t1 $\approx$ t2 if $\exists$ S. S (t1) = S (t2)

- Most General Unifier S: A substitution S:
    - S (t1) = S (t2)
    - $\forall$S'. S' (t1) = S' (t2) ➔ $\forall$t. S'(t) < S(t).

Polymorphic Type checking Translation Scheme

E -> E1 (E2) { p := mkleaf(newtypevar); unify (E1.type, mknode('->', E2.type, p);
E.type = p}

E -> E1, E2 {E.type := mknode('x', E1.type, E2.type); }

E -> id { E.type := fresh (id.type) }

fresh (t): replaces bound vars in t by fresh vars. Returns pointer to a node representing result.type.

fresh(∀ α.pointer(α) -> α) = pointer(α1) -> α1.

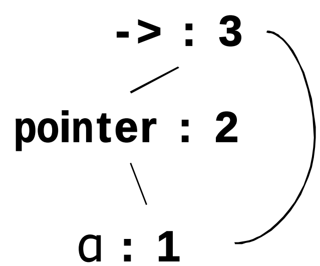
unify (m, n): unifies expressions represented by m and n.

- Side-effect: keep track of substitution
- Fail-to-unify: abort type checking.

PType Checking Example

Given: derefo (derefi (q))

q = pointer (pointer (int))



Bottom Up: fresh (∀α. Pointer(α) -> α)