Analog Communication Systems

EC-413-F

Lecture No 4,5

Topics To be Covered

- Noise in CW Modulation System

Noise in CW Modulation System

- 5.1 Introduction
  - \- Receiver Noise (Channel Noise) :
  - additive, White, and Gaussian 으로 가정

- 5.2 Receiver Model
- 1. RX Model

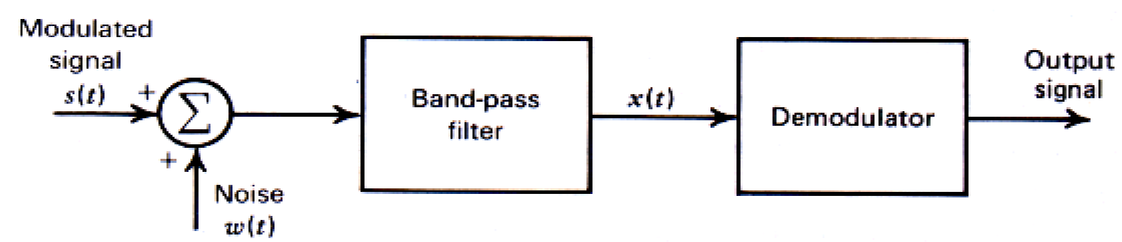
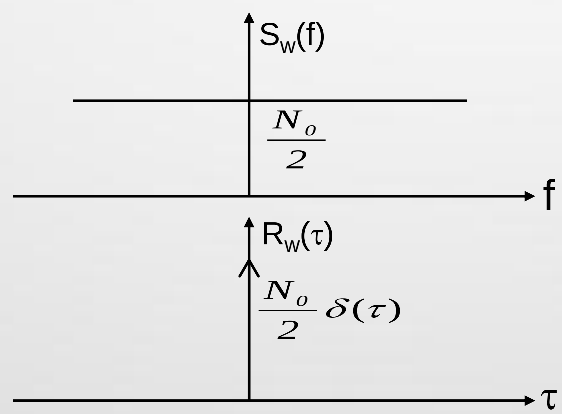


Figure 5.1 Noisy receiver model.

\- w(t) : additive, white, and Gaussian, power spectral density $= \frac{N_0}{2}$



- $N_o = KT_e$ where K = Boltzmann's constant
- T_e = equivalent noise Temp.
- Average noise power per unit bandwidth

- - Band Pass Filter (Ideal case)

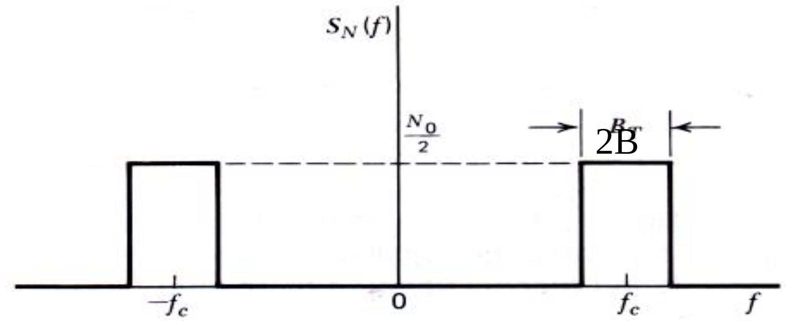
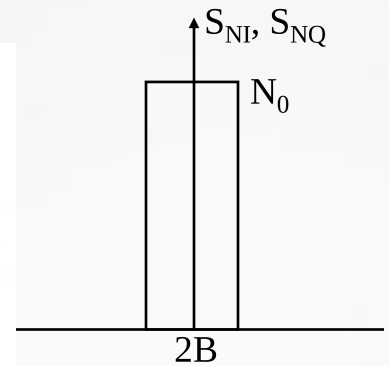
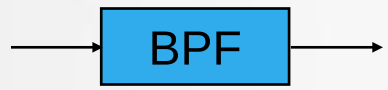


Figure 5.2 Idealized characteristic of band-pass filtered noise.



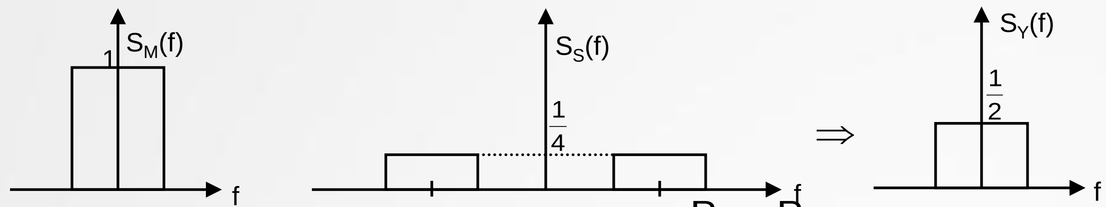


- w(t) n(t)
-
- - filtered noise as narrow-band noise
- $n(t) = n_I(t)\cos(2\pi f_C t) - n_Q(t)\sin(2\pi f_C t)$
  - where $n_I(t)$ is inphase, $n_Q(t)$ is quadrature component

- - ∴ filtered signal x(t)
- $x(t) = s(t) + n(t)$
-
- - Average Noise Power = $N_o B_T$
- $$-\ (SNR)_I = \frac{\text{average power of the modulated signal } s(t)}{\text{average power of the filtered noise } n(t)}$$

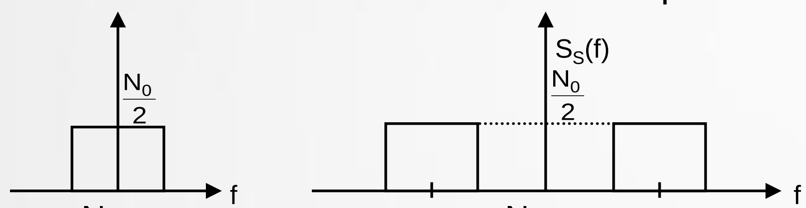
$$-\ (SNR)_O = \left.\frac{\text{average power of the demodulate d message signal}}{\text{average power of the noise}}\right|_{\text{receiver output}}$$

- DSB



$m(t) \rightarrow P$ $\quad m(t)\cos(2\pi f_C t) \rightarrow 2 \times (\frac{P}{4}) = \frac{P}{2}$

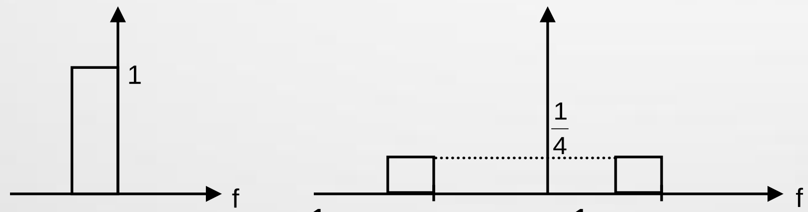
- $s(t) = m(t)\cos(2\pi fCt + \Theta) \rightarrow S_S(f) = \frac{1}{4}[S_M(f - f_C) + S_M(f + f_C)]$



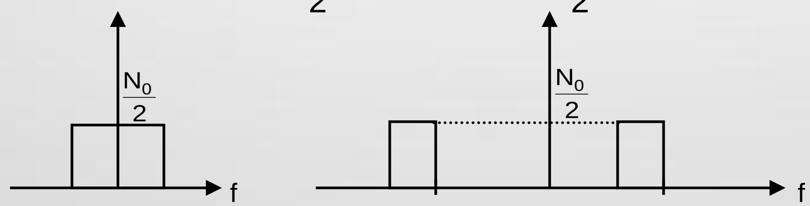
$2W\frac{N_0}{2} = WN_0$ $\quad 4W\frac{N_0}{2} = 2WN_0$

$\frac{1}{2}m(t) + \frac{1}{2}n_I(t)$

- SSB



$\frac{1}{2}m(t)\cos(2\pi f_C t) + \frac{1}{2}\hat{m}(t)\sin(2\pi f_C t) \rightarrow \frac{1}{4}\left(\frac{P}{2} + \frac{P}{2}\right) = \frac{P}{4}$

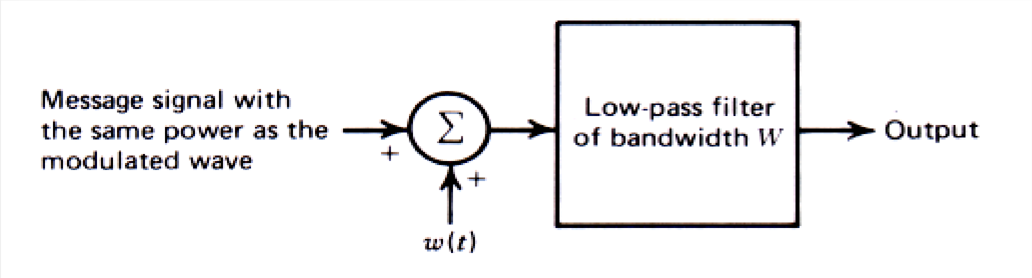


$2W\frac{N_0}{2} = WN_0$ $\quad 2W\frac{N_0}{2} = WN_0$

$\frac{1}{4}m(t) + \frac{1}{2}n_I(t)\cos(\pi Wt) + \frac{1}{2}n_Q(t)\sin(\pi Wt)$

- \- s(t) by each system has the same average power
- \- noise w(t) has the same average power measured in the message
- BW =W

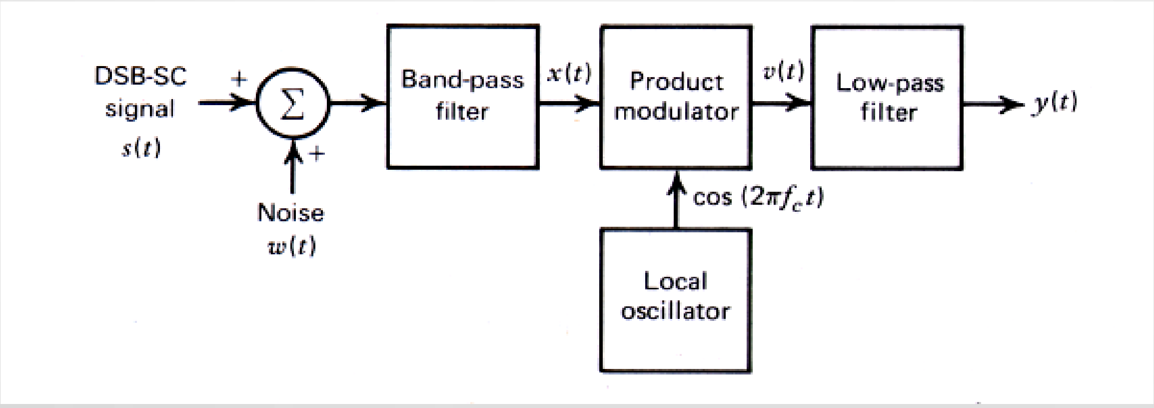
- 1) Channel SNR



$$(SNR)_C = \left. \frac{\text{average power of the modulated signal}}{\text{average power of the noise in the message BW}} \right|_{\text{at receiver input}}$$

- 2) $$\text{Figure of merit} = \frac{(SNR)_O}{(SNR)_C}$$

- Noise in DSB-SC Receivers

- 

- $s(t) = CA_C \cos(2\pi f_C t) m(t)$

  where C : scaling factor

  Power spectral density of m(t) : $S_M(f)$

  W : message bandwidth

- Average signal power

  $$P = \int_{-W}^{W} S_M(f)\, df$$

- Average power of s(t) $= \frac{C^2 A_C^2 P}{2}$

- Average noise power $= 2W \times \frac{N_0}{2} = WN_0$

  (baseband)

- $(SNR)_{C,DSB} = \frac{C^2 A_C^2 P}{2WN_0}$

- $x(t) = s(t) + n(t)$

  $= CA_C \cos(2\pi f_C t) m(t) + n_I(t) \cos(2\pi f_C t) - n_Q(t) \sin(2\pi f_C t)$

- $v(t) = x(t) \cos(2\pi f_C t)$

  $= \frac{1}{2} CA_C m(t) + \frac{1}{2} n_I(t) + \frac{1}{2}[CA_C m(t) + n_I(t)] \cos(4\pi f_C t) - \frac{1}{2} A_C n_Q(t) \sin(4\pi f_C t)$

$\therefore y(t) = \frac{1}{2} CA_C m(t) + \frac{1}{2} n_I(t)$

- 2. $(SNR)_C$

- Average signal power $= \dfrac{C^2 A_C^2 P}{4}$
- Average noise power $= \dfrac{1}{4}(2W)N_0 = \dfrac{1}{2} W N_0$ (passband)

  $\because \text{Power}(n_I(t)) = \text{Power of band pass filtered noise } n(t) = 2WN_0$
- $\therefore (SNR)_O = \dfrac{C^2 A_C^2 P/4}{W N_0/2} = \dfrac{C^2 A_C^2 P}{2 W N_0}$

  $\therefore$ Figure of merit

$$\left.\frac{(SNR)_O}{(SNR)_C}\right|_{DSB-SC} = 1$$

Noise in SSB Receivers

- - SSB Modulated wave

$$s(t) = \frac{1}{2} C A_C \cos(2\pi f_C t) m(t) + \frac{1}{2} C A_C \sin(2\pi f_C t) \hat{m}(t)$$

- $m(t)$ and $\hat{m}(t)$ are othogonal, $E[m(t)] = 0$

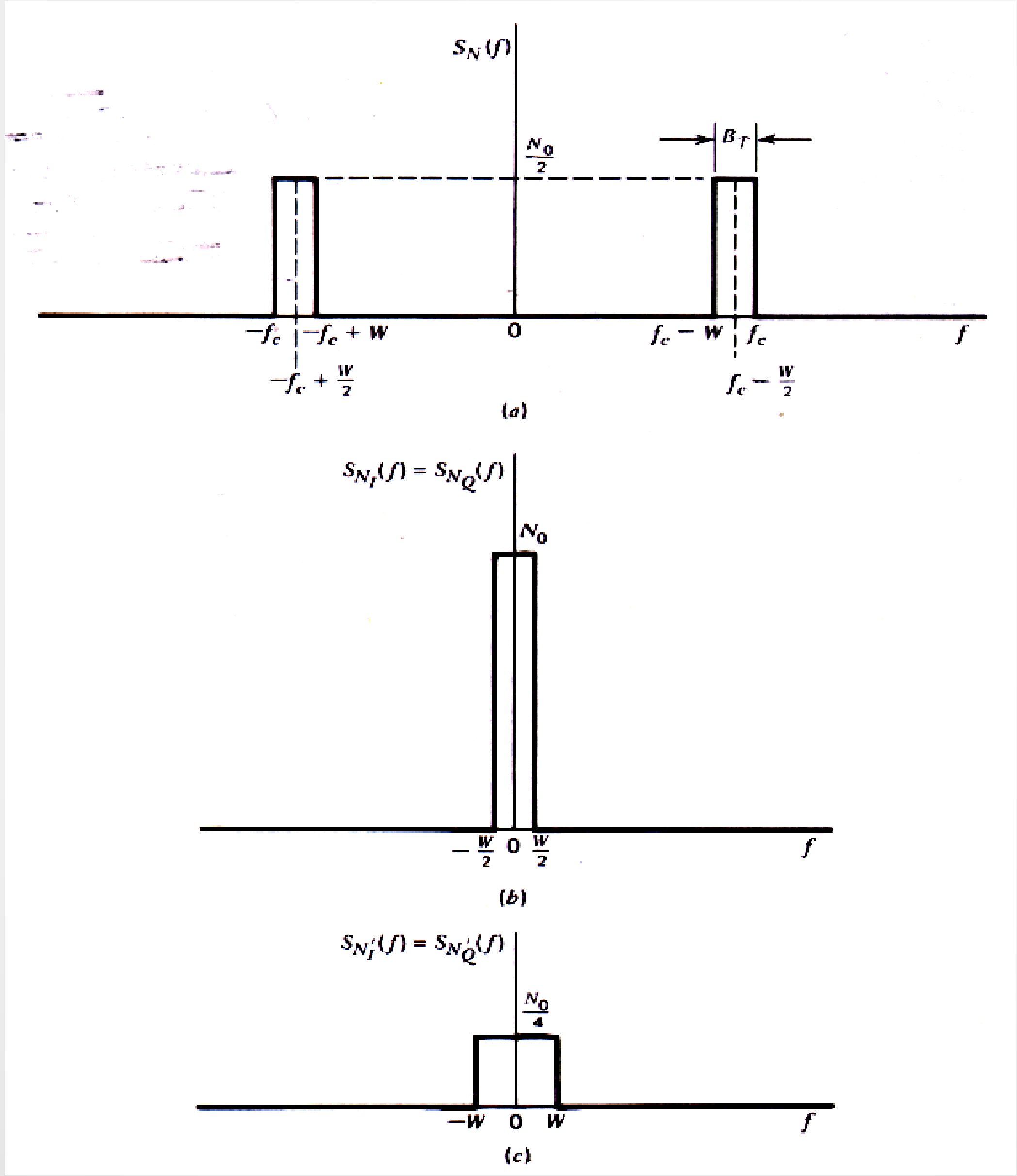
  $\Rightarrow$ $m(t)$ and $\hat{m}(t)$ are uncorrelated

  $\Rightarrow$ their power spectral densities are additive
- $m(t)$ and $\hat{m}(t)$ has the same power spectral density

- Message power $= \dfrac{C^2 A_C^2}{4} \dfrac{P}{2} + \dfrac{C^2 A_C^2}{4} \dfrac{P}{2} = \dfrac{C^2 A_C^2 P}{4}$

  (half of DSB)
- Average noise power $= W N_0$ ($\because$ message BW 안 의 Noise) (baseband)

$$(SNR)_{C,SSB} = \frac{C^2 A_C^2 P}{4 W N_0}$$



$$n(t) = n_I(t)\cos\left(2\pi(f_C - \frac{W}{2})t\right) - n_Q(t)\sin\left(2\pi(f_C - \frac{W}{2})t\right)$$

- Combined output

$$y(t) = \frac{1}{4}CA_C m(t) + \frac{1}{2}n_I(t)\cos(\pi Wt) + \frac{1}{2}n_Q(t)\sin(\pi Wt)$$

- Average signal power $= \frac{1}{16}C^2A_C^2P$
- Average noise power $= \frac{1}{4}\frac{WN_0}{2} + \frac{1}{4}\frac{WN_0}{2} = \frac{1}{4}WN_0$ (passband)
- $(SNR)_{O,SSB} = \frac{C^2A_C^2P}{4WN_0}$
- Figure of merit $\left.\frac{(SNR)_O}{(SNR)_C}\right|_{SSB} = 1$ same as DSB - SC

5.4 Noise in AM Receiver

- AM signal

$$s(t) = A_C[1 + k_a m(t)]\cos(2\pi f_C t)$$

- Average signal power $= A_C^2(1 + k_a^2P)/2$
- Average noise power $= WN_0 \leftarrow (2W \times \frac{N_0}{2})$

$$(SNR)_{C,AM} = \frac{A_C^2(1 + k_a^2P)}{2WN_0}$$

- Filtered signal

$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= [A_C + A_C k_a m(t) + n_I(t)]\cos(2\pi f_C t) - n_Q(t)\sin(2\pi f_C t) \end{aligned}$$

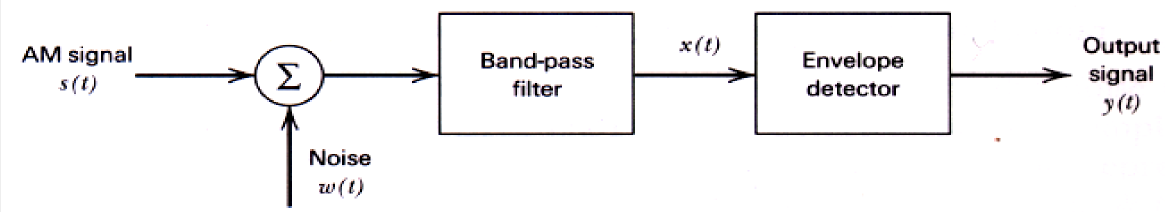


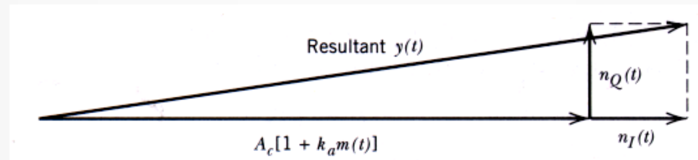
Figure 5.6 Noisy model of AM receiver.

$$y(t) = \text{envelop of } x(t)$$

$$= \left\{[A_C + A_C k_a m(t) + n_I(t)]^2 + n_Q^2(t)\right\}^{\frac{1}{2}}$$

Assume $A_C[1+k_a m(t)] \gg n_I(t), n_Q(t)$

$$y(t) \cong A_C + A_C k_a m(t) + n_I(t)$$



- $(SNR)_{O,AM} = \dfrac{A_C^2 k_a^2 P}{2WN_0}$

조건 $\begin{cases} \text{Avg carrier power} > \text{Avg noise power} \\ k_a \le 1 \end{cases}$

- Figure of merit $\left.\dfrac{(SNR)_O}{(SNR)_C}\right|_{AM} \cong \dfrac{k_a^2 P}{1+k_a^2 P} < 1$

- ex) Single Tone Modulation

$$m(t) = A_m \cos(2\pi f_m t) \rightarrow P = \frac{A_m^2}{2}$$

$$s(t) = A_C[1+\mu \cos(2\pi f_m t)]\cos(2\pi f_C t) \quad \text{where } \mu = k_a A_m$$

$$\left.\frac{(SNR)_O}{(SNR)_C}\right|_{AM} = \frac{\frac{1}{2}k_a^2 A_m^2}{1+\frac{1}{2}k_a^2 A_m^2} = \frac{\mu^2}{2+\mu^2}$$

$$\text{if } \mu = 1, \quad \text{F.O.M} = \frac{1}{3}(\text{max})$$

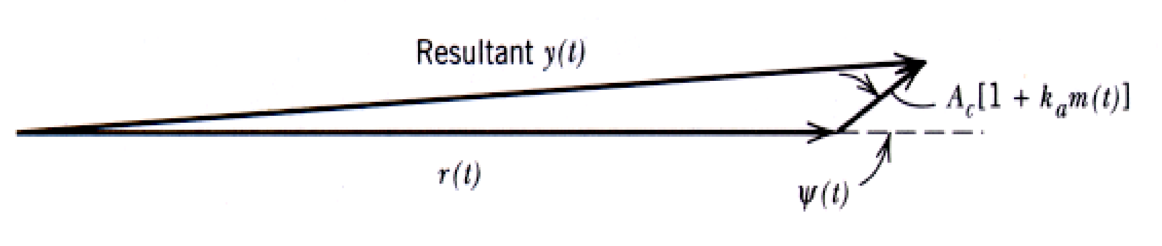
Threshold Effect

- Carrier-to-noise < 1
- narrow-band noise n(t)

$$n(t) = r(t)\cos[2\pi f_c(t) + \psi(t)]$$

where $r(t)$ is envelope, $\psi(t)$ is phase

$$x(t) = s(t) + n(t)$$
$$= A_c[1 + k_a m(t)]\cos(2\pi f_c t) + r(t)\cos(2\pi f_c t + \psi(t))$$



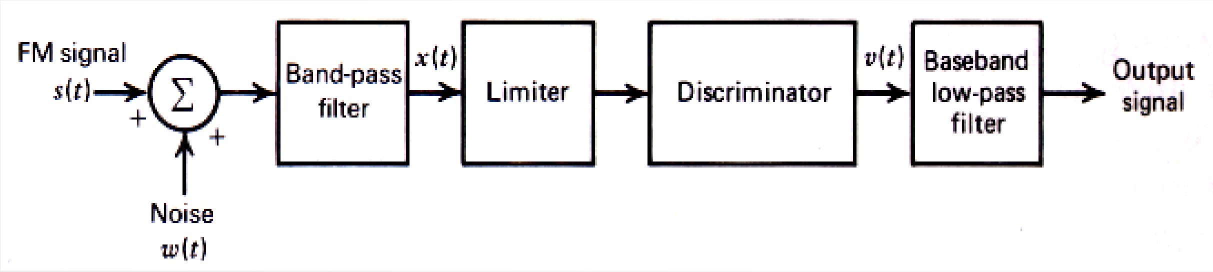
$$y(t) \cong r(t) + A_c[1 + k_a m(t)]\cos[\psi(t)]$$
$$\cong r(t) + A_c\cos[\psi(t)] + A_c k_a m(t)\cos[\psi(t)]$$

where $\psi(t)$ is uniformly distributed over $[0, 2\pi]$

$\Rightarrow$ complete loss of information

- Threshold Effect : loss of message in an envelope detector that
- operates at a low CNR.
-

- Noise in FM Receivers



  - w(t) : zero mean white Gaussian noise with psd = $N_0/2$
  - s(t) : carrier =f_c, BW = B_T 즉 $(f_C \pm B_T/2)$
- - BPF : $[f_C - B_T/2 \sim f_C + B_T/2]$
- - Amplitude limiter : remove amplitude variations
- by clipping and BPF
- - Discriminator
  - slope network or differentiator : varies linearly with frequency
  - envelope detector
- - Baseband LPF :
- - FM signal

$$s(t) = A_C \cos[2\pi f_C t + 2\pi k_f \int_0^t m(t)dt]$$

$$\phi(t) = 2\pi k_f \int_0^t m(t)dt$$

$$s(t) = A_C \cos[2\pi f_C t + \phi(t)]$$

$$n(t) = n_I(t)\cos(2\pi f_C t) + n_Q(t)\sin(2\pi f_C t) \\ = r(t)\cos[2\pi f_C t + \psi(t)]$$

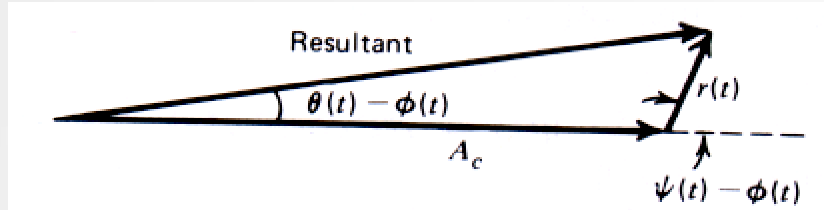
- - Filtered noise n(t)

where

$$\begin{cases} r(t) = \sqrt{(n_I(t))^2 + (n_Q(t))^2} \\ \psi(t) = \tan^{-1}\left[\dfrac{n_Q(t)}{n_I(t)}\right] \end{cases}$$

$$\therefore x(t) = s(t) + n(t)$$

$$= A_C \cos[2\pi f_C t + \phi(t)] + r(t)\cos[2\pi f_C t + \psi(t)]$$



$$\text{where } \Theta(t) = \phi(t) + \tan^{-1}\left\{ \frac{r(t)\sin[\psi(t) - \phi(t)]}{A_C + r(t)\cos[\psi(t) - \phi(t)]} \right\}$$

Assume $A_C \gg r(t)$

$$\Theta(t) \cong \phi(t) + \frac{r(t)}{A_C}\sin[\psi(t) - \phi(t)]$$

$$\cong 2\pi k_f \int_0^t m(t)dt + \frac{r(t)}{A_C}\sin[\psi(t) - \phi(t)]$$

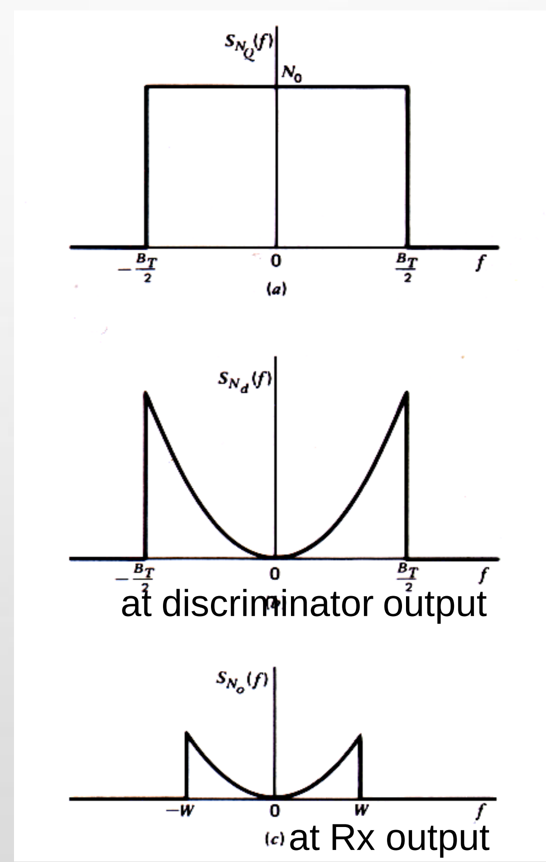
$$v(t) = \frac{1}{2\pi}\frac{d\Theta(t)}{dt} \cong k_f m(t) + n_d(t)$$

where

$$n_d(t) = \frac{1}{2\pi A_C}\frac{d}{dt}\{r(t)\sin[\psi(t) - \phi(t)]\}$$

$$\cong \frac{1}{2\pi A_C}\frac{d}{dt}\{r(t)\sin[\psi(t)]\}$$

$$\therefore n_d(t) = \frac{1}{2\pi A_C}\frac{dn_Q(t)}{dt}$$



- Pre-emphasis and de-emphasis in FM

  - P.S.D. of noise at FM Rx output

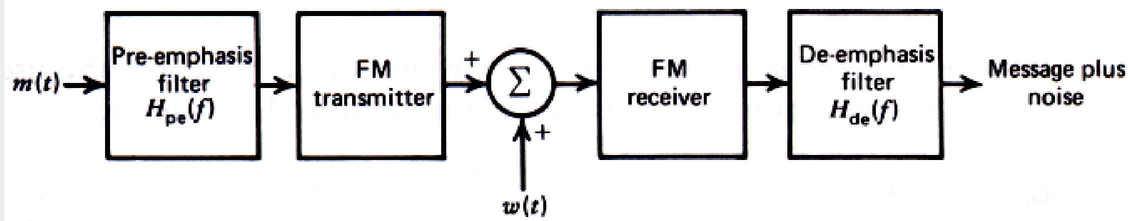
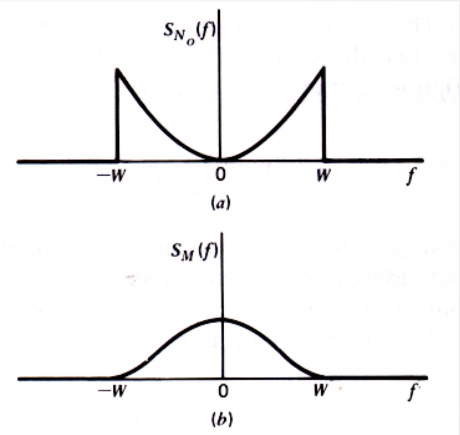
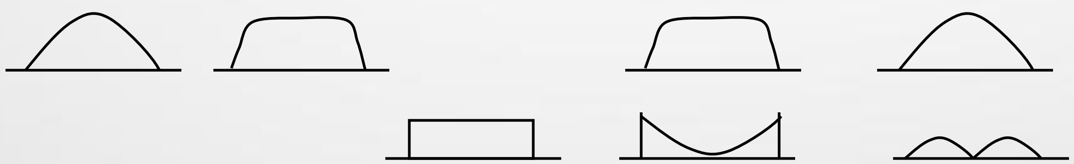


Figure 5.17 Use of pre-emphasis and de-emphasis in an FM system.



signal

noise



$$H_{de}(f) = \frac{1}{H_{pe}(f)}, \qquad -W \le f \le W$$

P.S.D of noise $n_d(t)$ at the discriminator output

$$S_{Nd}(f) = \begin{cases} \frac{N_0 f^2}{A_C^2} & |f| \le \frac{B_T}{2} \\ 0 & \text{otherwise} \end{cases}$$