NETWORK THEORY

LECTURE 2

SECTION B

TOPIC COVERED :NETWORK FUNCTIONS FOR ONE-PORT AND TWO PORT

NETWORKS, POLES AND ZEROS OF NETWORK FUNCTIONS

NETWORK FUNCTION

The network function *H*(*s*) is defined as the ratio of Laplace transform of the response (output) to
the Laplace transform of the excitation (Input), provided that all initial conditions of the network
are set to zero.

$$H(s) = \left.\frac{Y(s)}{X(s)}\right|_{\text{all initial conditions} = 0}$$

We define two different sets of network functions
(*i*) Driving point junction
(*ii*) Transfer function

Network functions are called driving point functions if the excitation (input) and the response (output) are defined at the same pair of terminals.

- There are two types of driving point functions.

  (*i*) Driving-point impedance function

  (*ii*) Driving-point admittance function

Note: These two functions are given one common name immittance function.

- For the input port, the driving-point impedance function is defined as

$$Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)}$$

- The driving-point admittance function is defined as the reciprocal of the driving-point impedance function.

$$Y_{in}(s) = \frac{1}{Z_{in}(s)} = \frac{I_{in}(s)}{V_{in}(s)}$$

- The output driving point functions are defined in a similar way.

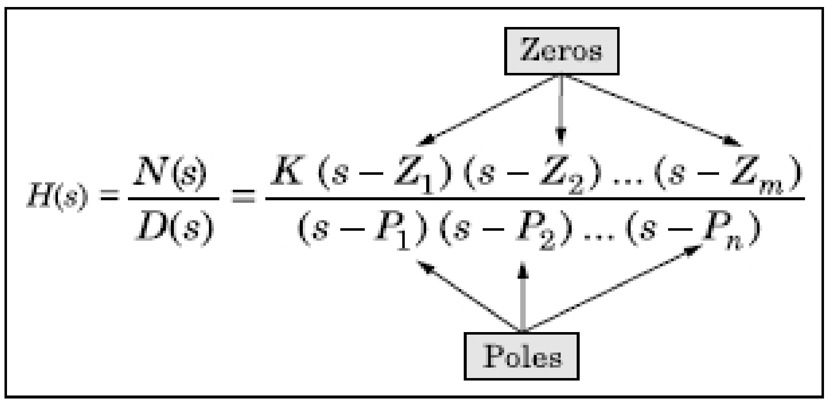
POLES AND ZEROS OF NETWORK FUNCTIONS

The network function $H(s)$ of a lumped linear network is the ratio-of two polynomial of s with real coefficients. In most general form of $H(s)$ is given by

$$H(s) = \frac{N(s)}{D(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}$$

The network function $H(s)$ can be written in the factored form as

$$H(s) = \frac{N(s)}{D(s)} = \frac{K\,(s - Z_1)\,(s - Z_2)\dots(s - Z_m)}{(s - P_1)\,(s - P_2)\dots(s - P_n)}$$



where $Z_1, Z_2 \dots Z_m$ and $P_1, P_2 \dots P_n$ are called complex frequencies. The constant multiplier

$K = \frac{a_m}{b_n}$ is called the scale factor.

- Zero : A zero of the system function $H(s)$ is defined as the value of s which makes $H(s) = 0$.
- Pole : A pole of $H(s)$ is defined as the value of s which makes the denominator of $H(s)$ approach zero or which makes $H(s)$ approach to infinity.

RESTRICTIONS ON POLE AND ZERO LOCATIONS FOR TRANSFER FUNCTIONS

(With common factor in $N(s)$ and $D(s)$ must cancelled)

(*i*) If the poles and Zeros are complex or imaginary then they must occur in complex conjugate pairs.

(*ii*) The coefficients of different terms in $N(s)$ and $D(s)$ must be real and those for $D(s)$ must be positive.

(*iii*) The real part of the pole must be negative or zero. If the real part is zero, then the pole must be simple. This includes the origin.

(*iv*) The degree of $N(s)$ may be as small as zero, independent of the degree of $D(s)$.

(*v*) The polynomials $D(s)$ should not have any missing terms between those of the highest and the lowest degree, unless all even or all odd terms are missing.

(*vi*) For $G_{21}(s)$ and $\alpha_{21}(s)$, the maximum degrees of $N(s)$ is the degree of $D(s)$.

(*vii*) For $Z_{21}(s)$ and $Y_{21}(s)$, the maximum degree of $D(s)$ plus one.

(*viii*) The polynomials $N(s)$ may have missing terms between those of the highest and lowest degree.

RESTRICTIONS ON POLE AND ZERO LOCATIONS FOR DRIVING POINT IMMITTANCE FUNCTION

Functions (With common factors in $N(s)$ and $D(s)$ cancelled)

(*i*) If the poles or zeros are complex or imaginary they must occur in complex conjugate pairs.

(*ii*) The coefficients of different terms in $N(s)$ and $D(s)$ must be real and positive.

(*iii*) The real part of all poles and zeros must be either negative or zero.

(*iv*) If the real part of any pole or zero is equal to zero then that pole or zero must be simple

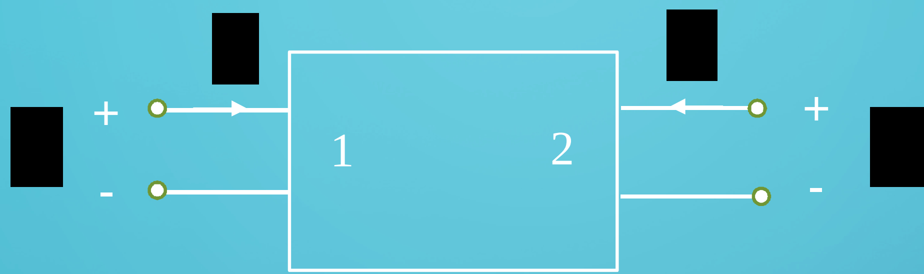
(*v*) The degree of $N(s)$ and $D(s)$ may differ by 0 or 1 only.

(*vi*) The lowest degree terms in $N(s)$ and $D(s)$ may differ in degree by almost 1.

(*vii*) The polynomial of $N(s)$ and $D(s)$ cannot have missing terms between those of highest and lowest degree of s, unless all even degree or all odd degree terms are missing.

Two-Port Networks

Consider a general 2-port linear network:



In terms of *Z*-parameters, we have (from superposition)

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Impedance (*Z*) matrix

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \Rightarrow [V] = [Z][I]$$

Elements of Z-Matrix: Z-Parameters

(open-circuit parameters
)

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k = 0 \quad k \neq j}$$

Port 2 open circuited

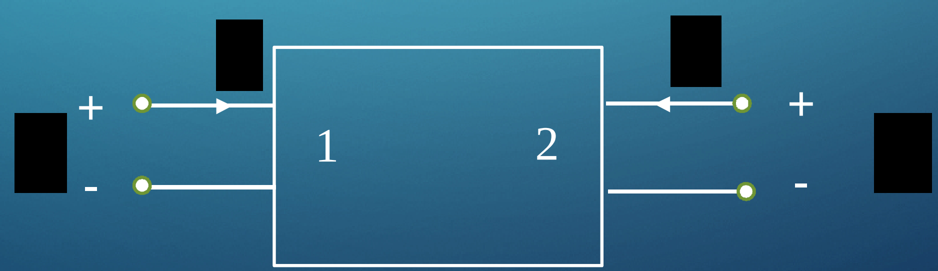
$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2 = 0}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = 0}$$

Port 1 open circuited

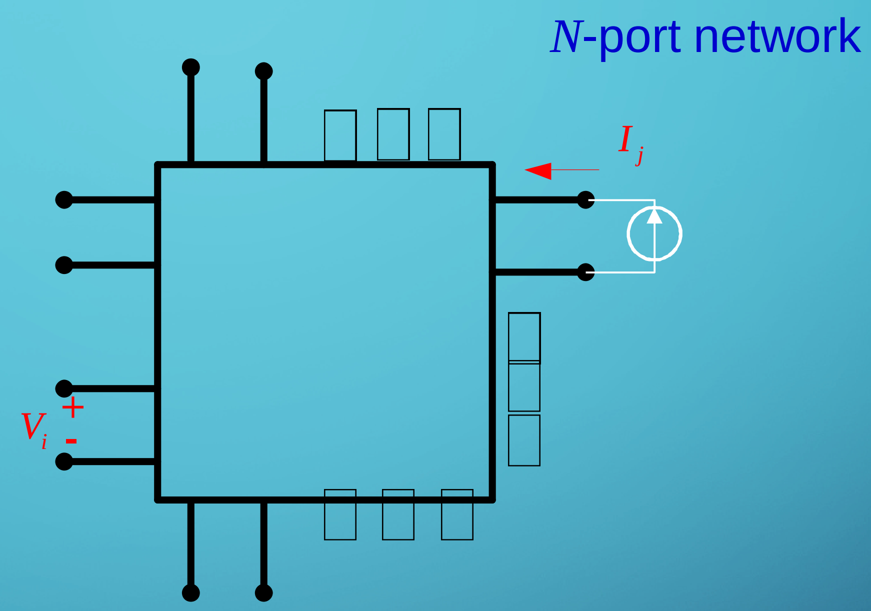
$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1 = 0}$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1 = 0}$$



Z-Parameters (cont.)

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k = 0 \quad k \neq j}$$



We inject a current into port j and measure the voltage (with an ideal voltmeter) at port i. All ports are open-circuited except j.

Z-Parameters (cont.)

Z-parameters are convenient for <u>series</u> connected networks.

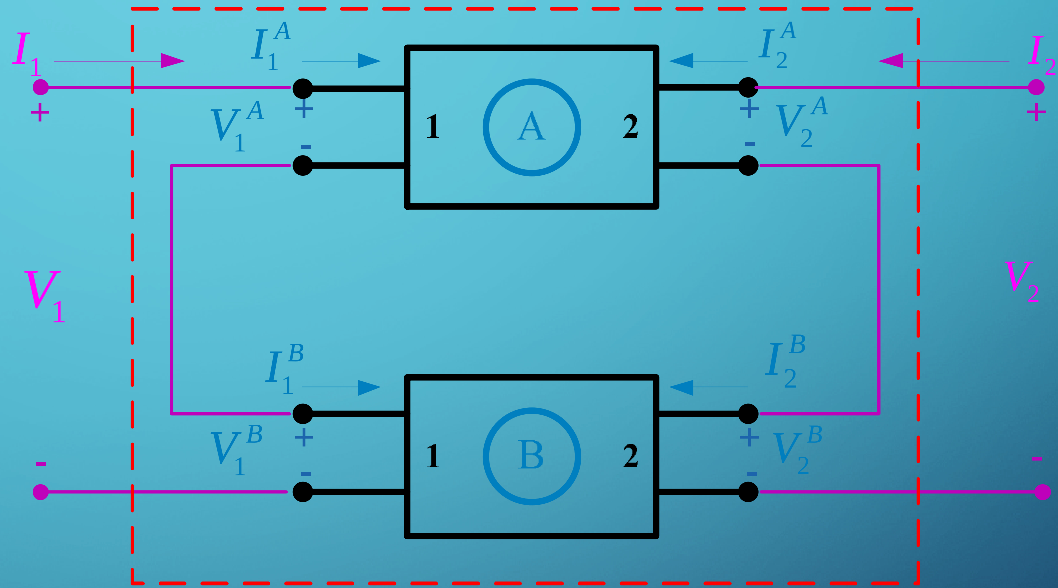
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1^A \\ V_2^A \end{bmatrix} + \begin{bmatrix} V_1^B \\ V_2^B \end{bmatrix}$$

$$= \left[Z^A\right]\left[I^A\right] + \left[Z^B\right]\left[I^B\right]$$

$$= \left(\left[Z^A\right] + \left[Z^B\right]\right)[I]$$

$$= \left(\left[Z^A\right] + \left[Z^B\right]\right)\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \left[Z^A + Z^B\right]\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11}^A + Z_{11}^B & Z_{12}^A + Z_{12}^B \\ Z_{21}^A + Z_{21}^B & Z_{22}^A + Z_{22}^B \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Series $\Rightarrow$ $I_1 = I_1^A = I_1^B$

$I_2 = I_2^A = I_2^B$

Admittance (Y) Parameters

Consider a 2-port network:

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Admittance matrix

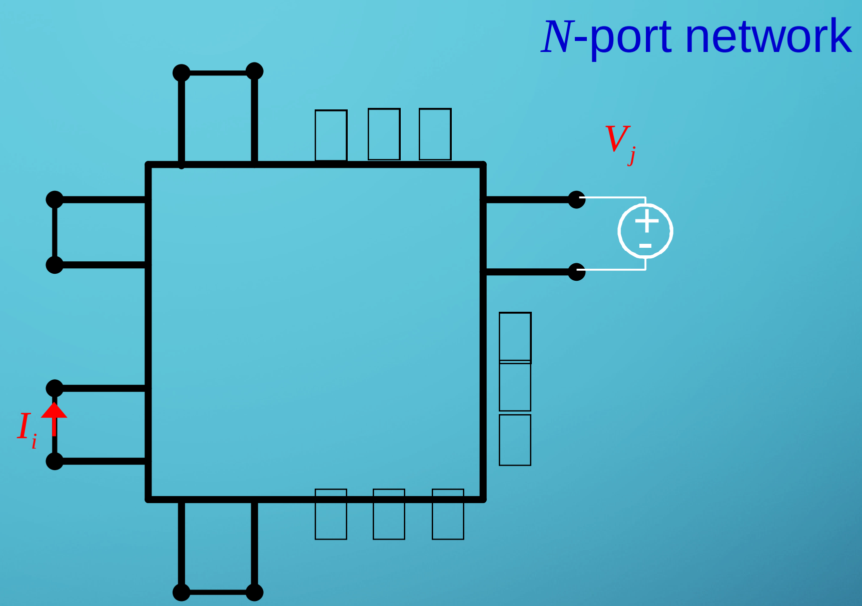
or $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow [I] = [Y][V]$$

$$Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_k = 0 \quad k \neq j}$$

Short-circuit parameters

Y-Parameters (cont.)

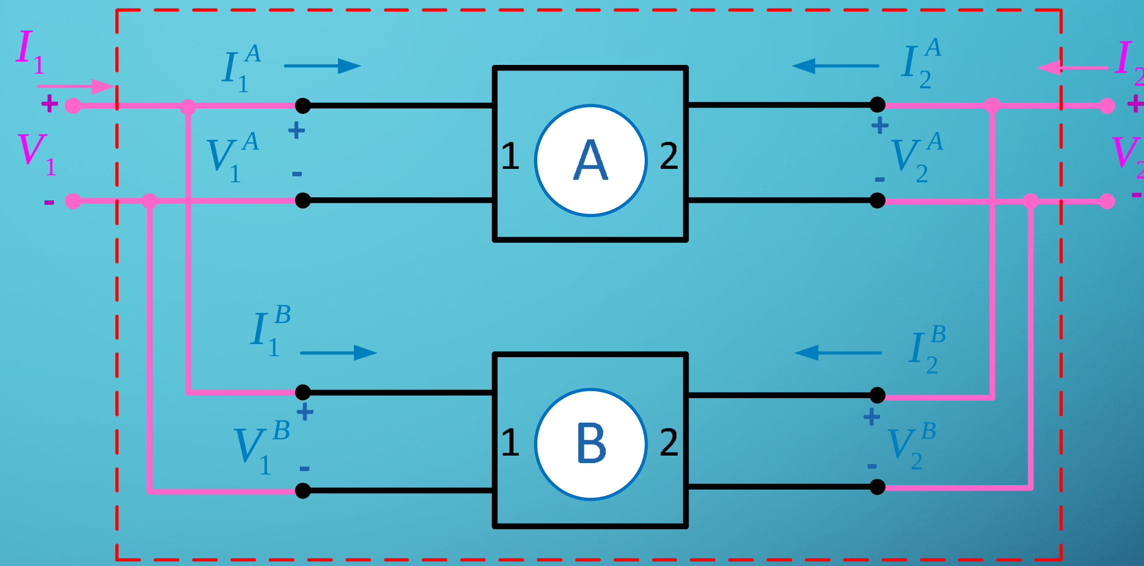
$$Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_k = 0 \quad k \neq j}$$



We apply a voltage across port j and measure the current (with an ideal current meter) at port i. All ports are short-circuited except j.

Admittance (Y) Parameters

Y-parameters are convenient for parallel connected networks.



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1^A \\ I_2^A \end{bmatrix} + \begin{bmatrix} I_1^B \\ I_2^B \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11}^A + Y_{11}^B & Y_{12}^A + Y_{12}^B \\ Y_{21}^A + Y_{21}^B & Y_{22}^A + Y_{22}^B \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Parallel $\Rightarrow V_1 = V_1^A = V_1^B$

$V_2 = V_2^A = V_2^B$

Admittance (Y) Parameters

Relation between [Z] and [Y] matrices:

$$[V] = [Z][I]$$

$$[I] = [Y][V]$$

Hence $$[V] = [Z]\left([Y][V]\right)$$
$$= \left([Z][Y]\right)[V]$$

$$[Z][Y] = [U] = \text{Identity Matrix}$$

Therefore $$[Y] = [Z]^{-1}$$